



Independent Schools
Examinations Board

**COMMON ENTRANCE EXAMINATION AT 11+ AND 13+
COMMON ACADEMIC SCHOLARSHIP EXAMINATION AT 13+**

MATHEMATICS SYLLABUS

(Revised September 2008 for first examination in Spring 2010)

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INTRODUCTION

The curriculum which pupils follow at school should not be restricted by this examination syllabus which is intended to be used simply as a guide. It is presumed that, as in all good practice, teachers will, where it is appropriate for their pupils, teach beyond the syllabus. The syllabus reflects the initiatives in both the National Curriculum and the National Numeracy Strategy *Framework for teaching mathematics*. It is assumed that all teachers of mathematics will be familiar with the relevant publications. It is expected that pupils will be familiar with the skills and knowledge of National Curriculum key stage 1 and the National Numeracy Strategy *Framework* for the early years.

AIMS

A course leading to these examinations should:

- (i) encourage breadth of experience in the development of mathematical skills without in any way prejudicing thorough grounding;
- (ii) encourage the development of investigative thinking and the application of mathematical knowledge to unfamiliar problems.

ASSESSMENT OBJECTIVES

11+

- AO1 Candidates should be familiar with the skills and knowledge required at the end of key stage 2.
- AO2 Topics are centred at National Curriculum levels 4 and 5, based on the yearly teaching programmes up to year 6 of the National Numeracy Strategy, with particular reference to the year 5 and year 6 key objectives.

13+

- AO1 Candidates should be familiar with most of the skills and knowledge of key stage 3.
- AO2 Topics are centred at (a) National Curriculum levels 4 and 5 with some elements of level 6 for lower-level candidates, (b) National Curriculum levels 5 and 6 with some elements of level 7 for standard-level candidates and (c) years 7, 8 and 9 of the National Numeracy Strategy.

SYLLABUS CONTENT

The key stage 2 and 3 programmes of study describe the skills, knowledge and understanding required.

During key stage 2, candidates build on the skills which they developed in key stage 1. They extend their competence and confidence with number so that they move from security in counting to security in calculating with all four operations. Candidates explore features of shape and space and develop their measuring skills in a range of contexts. They discuss and present their methods and reasoning, using a wider range of mathematical language, diagrams and charts.

During key stage 3, candidates build on the skills which they developed in key stage 2 and increasingly make connections between different aspects of mathematics. They extend their calculating skills to fractions, percentages and decimals. They begin to understand the importance of reasoning about proportion. They start to develop facility in the use of algebraic techniques and symbols. They study linear functions and their corresponding graphs. Candidates progress from a simple understanding of the features of shape and space to using definitions and reasoning to understand geometrical objects. They undertake practical data handling work, introducing a quantitative approach to probability. Candidates should work with increasing confidence and flexibility to solve unfamiliar problems, including word problems, and develop a positive attitude towards mathematics.

The syllabus content is based on attainment targets 2 (number and algebra), 3 (shape, space and measures) and 4 (handling data) of the National Curriculum. It is set out in columns on the following pages. The descriptions for levels 4–7 for each attainment target are printed in the first column. Topics printed in **bold** are additions to the National Curriculum. In the middle columns, shading indicates that a topic is not examined at that level. For 13+, in Level 1, questions will be set on topics from Levels 4 and 5 and only those in Level 6 marked with an asterisk; the letter E indicates that the topic will be examined only in Level 3 and in the Scholarship examination paper. Amplification of the National Curriculum descriptions and examples are given in the right-hand column.

ATTAINMENT TARGET 2: NUMBER AND ALGEBRA

Level 4	11+	13+	
<ul style="list-style-type: none"> Pupils use their understanding of place value to multiply and divide whole numbers by 10 or 100 			<p>There are 65 pages in a book. How many pages are there altogether in 100 such books?</p> <p>Answers may be decimals, e.g. $46 \div 10$</p>
<ul style="list-style-type: none"> In solving number problems, they use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 10×10 and quick derivation of corresponding division facts. 			
<ul style="list-style-type: none"> They use efficient written methods of addition and subtraction and of short multiplication and division. They add and subtract decimals to two places and order decimals to three places. 			<p>One piece of wood is 1.78m long; another is 0.48m longer. Calculate the length of the second piece.</p>
<ul style="list-style-type: none"> In solving problems with or without a calculator, they check the reasonableness of their results by reference to their knowledge of the context or to the size of the numbers. 			<p>A piece of string 12m long is cut into four equal pieces. How long is each piece?</p>
<ul style="list-style-type: none"> They recognise approximate proportions of a whole and use simple fractions and percentages to describe these. 			<p>Candidates should know that 8 articles out of 16 is 50% of the total.</p> <p>Find 60% of £500</p>
<ul style="list-style-type: none"> They recognise and describe number patterns and relationships, including multiple, factor and square. 			<p>Candidates should know the terms square, square number, cube and simple square roots such as those of 4, 9, 16, 25 etc.</p>
<ul style="list-style-type: none"> They begin to use simple formulae expressed in words. 			<p>Candidates should be able to solve puzzles such as 'When I double a number and add one, I get seventeen. What is the number?'</p> <p>Candidates may be required to substitute values into such a formula.</p>
<ul style="list-style-type: none"> They use and interpret co-ordinates in the first quadrant. 			
<ul style="list-style-type: none"> They understand the concept of a prime number. 			<p>Candidates should know that 1 is not prime.</p> <p>They should know prime numbers to 20</p>
Level 5	11+	13+	
<ul style="list-style-type: none"> Pupils use their understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000 			<p>Questions such as 200×300 and $400 \div 20$ will be included.</p> <p>Multiply 1.47 by 1000</p>
<ul style="list-style-type: none"> They order, add and subtract negative numbers in context and in the abstract. 			<p>The temperatures at 0600 on three consecutive days were: -2°C, -1°C, and -6°C.</p> <p>What is the difference between the lowest and the highest temperature?</p> <p>Candidates should know that $3 - (-2) = 5$</p>

<ul style="list-style-type: none"> They use all four operations with decimals to two places. 			<p>At 11+, multiplication and division of decimals will always be by an integer less than 10. They should be able to round to the nearest integer.</p> <p>At 13+, this should include rounding to 1 or 2 decimal places.</p>
<ul style="list-style-type: none"> They reduce a fraction to its simplest form by cancelling common factors and solve simple problems involving ratio and direct proportion. 			<p>$\frac{4}{12} = \frac{1}{3}$ ratios such as 3:1</p> <p>150 ml of concentrated juice will make 500 ml of orange squash. How much concentrated juice will be needed for 750 ml of squash?</p>
<ul style="list-style-type: none"> They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. 			<p>Calculate $\frac{1}{10}$ of 3 m or $\frac{3}{4}$ of 60 kg</p> <p>Find 70% of £5</p> <p>Interpretation of calculator displays may be included at 11+.</p>
<ul style="list-style-type: none"> They understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three-digit number by any two-digit number. 			<p>Multiplication only at 11+.</p> <p>A shopkeeper buys 13 fridges each costing £312 How much do they cost altogether?</p> <p>£210 is paid for 15 identical theatre tickets. What is the cost of one ticket?</p> <p>Candidates should be able to do division by factors.</p>
<ul style="list-style-type: none"> They check their solutions by applying inverse operations or estimating, using approximations. 			<p>Estimate the total cost of 11 ice creams, costing 39 pence each, by calculating 10×40</p>
<ul style="list-style-type: none"> They construct, express in symbolic form, and use simple formulae involving one or two operations. 			<p>The cost c of n items at 20 pence is $c = 20n$ pence.</p> <p>If $p = 2(a + b)$, find the value of p when $a = 4, b = 5$</p>
<ul style="list-style-type: none"> They use brackets appropriately. 			<p>It is assumed that candidates will be familiar with the order of operations.</p> <p>$3 \times 4 + 5 = 17$ $3(4 + 5) = 27$</p>
			<p>This will include familiarity with simple expressions such as</p> <p>$x + 2, 3x, \frac{x}{2}, x^2, 3(x + 2)$</p>
			<p>Collect like terms, e.g.</p> <p>$a + b + a + b + b = 2a + 3b$ $x + 5 + 2x + 4 = 3x + 9$</p>
			<p>Multiply out brackets in straightforward examples, such as</p> <p>$3(x + 4) = 3x + 12$ $2(a + b) = 2a + 2b$</p>

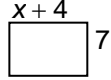
			<p>Candidates should be able to take out a common factor in simple cases, e.g.</p> $2a + 4b = 2(a + 2b)$ $3x + 12 = 3(x + 4)$ $x^2 + 2x = x(x + 2)$ <p>Simplify numerical and algebraic fractions, such as</p> $\frac{4x}{12} = \frac{x}{3}$ $\frac{3c}{c} = 3$
<ul style="list-style-type: none"> They use and interpret co-ordinates in all four quadrants. 			
<ul style="list-style-type: none"> They understand prime factors. 			Express 42 or 243 as a product of primes, using index notation.
Level 6	11+	13+	
<ul style="list-style-type: none"> Pupils order and approximate decimals when solving numerical problems and equations, using trial-and-improvement methods. 		E	Candidates should know that solving $x(5x + 2) = 16$ by 'trial and improvement' will involve completing a table of values to help identify the solution.
<ul style="list-style-type: none"> They are aware of which number to consider as 100 per cent, or a whole, in problems involving comparisons, and use this to evaluate one number as a fraction or percentage of another. 			A T-shirt marked at £18 was sold for £14.40 What percentage discount was given on the marked price?
<ul style="list-style-type: none"> They understand and use the equivalences between fractions, decimals and percentages, and calculate using ratios in appropriate situations. 		*	<p>Use ratio to adapt a recipe for 6 people to one for 8 people.</p> <p>Work out the selling price of trainers costing £24.50 when sold at a discount of 40%.</p> <p>Candidates should know that $\frac{50}{100}$, 0.5, 50%, are equivalent.</p> <p>Write $\frac{2}{7}$, 30% and 0.29 in ascending / descending order.</p>
<ul style="list-style-type: none"> They add and subtract fractions by writing them with a common denominator. They multiply and divide fractions, including easy mixed numbers. 		*	$\frac{1}{11} + \frac{4}{11} = \frac{5}{11}$ $\frac{1}{5} + \frac{2}{3}; \frac{3}{4} - \frac{1}{3}$
		E	$3\frac{1}{2} \times 1\frac{1}{3}; 1\frac{1}{2} - \frac{3}{5}$ $\frac{1}{2} \times \frac{1}{3}; \frac{1}{2} \text{ of } \frac{3}{4}; \frac{1}{2} \div \frac{1}{4}$
		E	$3\frac{1}{2} \times \frac{2}{3}; 3\frac{1}{2} \div 1\frac{1}{4}$

<ul style="list-style-type: none"> When exploring number sequences, they find and describe in words the rule for the next term or nth term of a sequence where the rule is linear. They describe the rule for such a sequence algebraically. 		E	<p>Find the 100th term in the sequence 3, 5, 7, ...</p> <p>Give the nth term of the above sequence as $2n + 1$</p>
<ul style="list-style-type: none"> They formulate and solve linear equations with whole-number coefficients and simple fractional coefficients. 		* * * E E E	<p>A rectangle has adjacent sides of length $(2x + 3)$ cm and $(5x - 1)$</p> <p>The perimeter is 46 cm. Find x.</p> <p>Questions set on algebraic equations will increase in difficulty in the approximate order shown.</p> $x + 5 = 12$ $3x = 12$ $3x - 4 = 8$ $8 = 3x - 4$ $3x - 4 = 8 - x$ $3(x - 4) = 9$ $4 - 3x = 9$ $\frac{1}{3}x = 2$ $\frac{1}{3}x - 4 = 2$ $\frac{3}{4}(3x - 4) = 6$
<ul style="list-style-type: none"> They represent mappings expressed algebraically, and use Cartesian co-ordinates for graphical representation, interpreting general features including quadratic functions. 		* E E	$y = 5, y = -2$ $y = x + 2, y = 3x - 1$ $y = x^2 + 2x$ <p>Plot points on the curve $y = x^2 + 2x$ and draw the line $y = x + 4$ on the same grid; find the co-ordinates of points of intersection.</p>
Level 7	11+	13+	
<ul style="list-style-type: none"> In making estimates, pupils round to one significant figure and multiply and divide mentally. 			<p>Estimate the value of $\frac{63 \times 87}{29}$ by writing it as $\frac{60 \times 90}{30}$</p>
<ul style="list-style-type: none"> They understand the effects of multiplying and dividing by numbers between 0 and 1 			<p>Work out $40 \times 0.3, 600 \div 0.3$</p>
<ul style="list-style-type: none"> They solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately. 			<p>Use a calculator to evaluate $\frac{732}{8 \times 17}$ or $\frac{17.6}{47.92 - 41.4}$</p>
<ul style="list-style-type: none"> They understand and use proportional changes, calculating the result of any proportional change using only multiplicative methods. 			<p>Candidates should know that if a motor car uses 40 litres of fuel on a 250 mile journey, it would be expected to use $40 \times \frac{300}{250}$ litres on a 300 mile journey.</p>

<ul style="list-style-type: none"> They find and describe in symbols the next term or nth term of a sequence where the rule is quadratic. 		E	Candidates will only be asked to substitute values.
<ul style="list-style-type: none"> They multiply two expressions of the form $(x + n)$ and simplify the corresponding quadratic expressions. 		E	This will be restricted to $x(x - 2) = x^2 - 2x$
<ul style="list-style-type: none"> They use algebraic and graphical methods to solve simultaneous linear equations in two variables. 		E	Solve $\left. \begin{array}{l} 2x + y = 10 \\ 3x - 2y = 8 \end{array} \right\}$
<ul style="list-style-type: none"> They solve simple inequalities. 		E	Solve $\frac{1}{2}(x - 1) > 3$ or $5 - 3x < 12$ Represent the solutions on a number line. Find the set of integers satisfying more than one inequality.

ATTAINMENT TARGET 3: SHAPE, SPACE AND MEASURES

Level 4	11+	13+	
<ul style="list-style-type: none"> Pupils make 3-D mathematical models, by linking given faces or edges; they draw common 2-D shapes in different orientations on grids. 			Candidates should be able to use a ruler, compasses, a protractor and set squares. They should be able to construct triangles reasonably accurately to the nearest degree.
<ul style="list-style-type: none"> They reflect simple shapes in a mirror line. 			Candidates should know that mirror lines will be parallel to the co-ordinate axes or at 45° to the grid lines.
<ul style="list-style-type: none"> They choose and use appropriate units and instruments, interpreting, with appropriate accuracy, numbers on a range of measuring instruments. 			Candidates should show an understanding of the relationships between units of measurement: km, m, cm, and mm; t(tonne), kg, g, mg; litre, cl, ml.
<ul style="list-style-type: none"> They find perimeters of simple shapes and find areas. 			Candidates should be able to compare rectangles with the same perimeters but different areas.
<ul style="list-style-type: none"> They find volumes by counting cubes. 			Candidates should be able to calculate the volumes of solids in isometric drawings.
<ul style="list-style-type: none"> They specify location by means of angle and distance. 			Candidates should understand the eight main points of the compass.
<ul style="list-style-type: none"> They understand and use the transformations rotation and translation; they identify congruent shapes and orders of rotational symmetry. 			Transformations include those drawn on a co-ordinate grid. Translations will be given in the form '3 units right, 4 units up'.
Level 5	11+	13+	
<ul style="list-style-type: none"> When constructing models, and when drawing or using shapes, pupils measure and draw angles to the nearest degree, and use language associated with angle. 			Candidates should be able to draw nets accurately, using given data. At 13+, reasons may be required to justify explanations of properties of shapes.
<ul style="list-style-type: none"> They know the angle sum of a triangle and the sum of angles at a point. 			Language associated with angle (e.g. acute, obtuse, reflex) may be used – see National Numeracy Strategy (NNS) vocabulary list. Knowledge of the sum of angles on a straight line is also assumed.

<ul style="list-style-type: none"> They identify all the symmetries of 2-D shapes. 			This includes the centre and order of rotational symmetry of a plane shape.
<ul style="list-style-type: none"> They know the rough metric equivalents of imperial units still in daily use and convert one metric unit to another. 			<p>Approximately how many miles are equivalent to 40 km?</p> <p>How many metres are there in 7.5 km?</p> <p>Units will be restricted to those listed in the NNS framework for year 7.</p>
<ul style="list-style-type: none"> They make sensible estimates of a range of measures in relation to everyday situations. 			Estimate the height of a door, the mass of a hockey ball, the capacity of a level teaspoon.
<ul style="list-style-type: none"> They understand and use the formula for the area of a rectangle. 			<p>At 13+, this might include examples such as</p> 
Level 6	11+	13+	
<ul style="list-style-type: none"> Pupils recognise and use common 2-D representations of 3-D objects. 			Candidates should be able to use isometric paper (supplied with the question paper).
<ul style="list-style-type: none"> They know and use the properties of quadrilaterals in classifying different types of quadrilateral. 			<p>Candidates should know the following: square, rectangle, trapezium, kite, parallelogram, rhombus.</p> <p>They should be able to recognise similar shapes.</p>
<ul style="list-style-type: none"> They solve problems using angle and symmetry properties of polygons and angle properties of intersecting and parallel lines, and explain these properties. 		*	<p>Candidates should know the names of regular polygons to decagon and be able to calculate interior and exterior angles of regular shapes.</p> <p>They should be able to solve problems using angle properties of intersecting and parallel lines.</p>
<ul style="list-style-type: none"> They devise instructions for a computer to generate and transform shapes and paths. 			<i>This topic will not be examined.</i>
<ul style="list-style-type: none"> They understand and use appropriate formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids when solving problems. 		*	<p>This includes areas of triangles and parallelograms.</p> <p>Level 1 candidates will be expected to answer questions involving volume of a cuboid and areas of triangles and parallelograms (formulae given)</p> <p>E Find the radius of a circle of given circumference or area.</p>
<ul style="list-style-type: none"> They enlarge shapes by a positive whole number scale factor. 			<p>The centre of enlargement will be given.</p> <p>Candidates should know that when the scale factor is 3, the area increases by a factor of 9</p>
<ul style="list-style-type: none"> They understand and use three-figure bearings to define direction. 			Candidates should be able to solve scale-drawing problems, such as those involving positions of ships.

Level 7	11+	13+	
<ul style="list-style-type: none"> Pupils understand and apply Pythagoras' theorem when solving problems in two dimensions. 		E	Candidates should be able to use Pythagoras' theorem in context in 2-D shapes to calculate one side of a right-angled triangle given the other two sides.
		E	Calculate the distance between two points given their 2-D co-ordinates.
<ul style="list-style-type: none"> They calculate lengths, areas and volumes in plane shapes and right prisms. 		E	Candidates should know the definition of a prism, and calculate the volume (but not surface area) of a right cylinder.
<ul style="list-style-type: none"> They enlarge shapes by a fractional scale factor, and appreciate the similarity of the resulting shapes. 			<i>This topic will not be examined.</i>
<ul style="list-style-type: none"> They determine the locus of an object moving according to a rule. 			<i>This topic will not be examined.</i>
<ul style="list-style-type: none"> They appreciate the imprecision of measurement and recognise that a measurement given to the nearest whole number may be inaccurate by up to one half in either direction. 			<i>This topic will not be examined.</i>
<ul style="list-style-type: none"> They understand and use compound measures, such as speed. 		E	Only speed will be tested. Questions may be set involving average speeds of multi-stage journeys.

ATTAINMENT TARGET 4: HANDLING DATA

Level 4	11+	13+	
<ul style="list-style-type: none"> Pupils collect discrete data and record them using a frequency table. 			Candidates should be familiar with tallying, pictograms, block graphs, bar charts, Carroll diagrams and Venn diagrams.
<ul style="list-style-type: none"> They understand and use the mode and range to describe sets of data. 			Candidates should understand how to read the mode from a frequency diagram, or find it from a list of values. They should be able to give the range of a set of data with values from 12 to 35 as 23
<ul style="list-style-type: none"> They group data, where appropriate, in equal class intervals, represent collected data in frequency diagrams and interpret such diagrams. 			Candidates should understand and be able to construct and use bar-line graphs and frequency diagrams with suitable class intervals. Questions may involve identifying the modal class.
<ul style="list-style-type: none"> They construct and interpret simple line graphs. 			This could include a patient's temperature graph, or a conversion graph.
<ul style="list-style-type: none"> They understand and use simple vocabulary associated with probability, including <i>fair</i>, <i>certain</i> and <i>likely</i>. 			Candidates should be able to distinguish between <i>fair</i> and <i>unfair</i> . They should be able to decide whether statements are <i>likely</i> , <i>unlikely</i> , <i>certain</i> or <i>impossible</i> . They should be able to determine from a results table whether or not a die is fair or biased.

Level 5	11+	13+	
<ul style="list-style-type: none"> Pupils understand and use the mean of discrete data. 			Calculate the mean (average) of the following eight test scores: 6, 6, 9, 4, 7, 10, 7, 7 At 11+ the number of items of data will be small – not more than 5
<ul style="list-style-type: none"> They understand and use the median of a set of data. 			Find the median value, including a strategy in cases where the number of items of data is even ('half-way' between the two 'middle' values).
<ul style="list-style-type: none"> They compare two simple distributions, using the range and one of the mode, median or mean. 			Compare marks in two exams shown on separate frequency diagrams. Suggest, with reasons, which exam was more difficult.
<ul style="list-style-type: none"> They interpret graphs and diagrams, including pie charts, and draw conclusions. 			
<ul style="list-style-type: none"> They understand and use the probability scale from 0 to 1. 			This includes numbers expressed as decimals, fractions or percentages.
<ul style="list-style-type: none"> They find and justify probabilities, and approximations to these, by selecting and using methods based on equally likely outcomes and experimental evidence, as appropriate. 			Candidates should understand the probability of a simple event happening, e.g. when rolling a fair die, the probability of scoring 5 is $\frac{1}{6}$
<ul style="list-style-type: none"> They understand that different outcomes may result from repeating an experiment. 			The experiment will involve a single event, where all possible outcomes can be listed.
Level 6	11+	13+	
<ul style="list-style-type: none"> Pupils collect and record continuous data, choosing appropriate equal class intervals over a sensible range to create frequency tables. 			<i>This topic will not be examined.</i>
<ul style="list-style-type: none"> They construct and interpret frequency diagrams. 			Class intervals will always begin and end with integers. Data will be discrete.
<ul style="list-style-type: none"> They construct pie charts. 			Either angles or percentages may be used as appropriate.
<ul style="list-style-type: none"> They draw conclusions from scatter diagrams, and have a basic understanding of correlation. 		*	
<ul style="list-style-type: none"> When dealing with a combination of two experiments, they identify all the outcomes, using diagrammatic, tabular or other forms of communication. 		*	
<ul style="list-style-type: none"> In solving problems, they use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1 			
Level 7	11+	13+	
<ul style="list-style-type: none"> Pupils draw a line of best fit on a scatter diagram, by inspection. 			They use the line to make predictions. <i>Apart from this, no level 7 handling data topic will be examined.</i>

SCHEME OF ASSESSMENT

At both stages of transfer, working and answers are to be shown on the question paper and in most questions, failure to show working may result in loss of marks.

11+

Candidates will be required to work one paper of 60 minutes' duration. All candidates should answer as many questions as possible. Whilst candidates should be familiar with the use of calculators, calculating aids may not be used in the examination.

13+

Candidates will be required to work two papers, one non-calculator and one calculator, each of 60 minutes' duration, and a mental test lasting up to 30 minutes. The non-calculator and calculator papers will be available at three levels: **Level 1**, **Level 2** and **Level 3**. Level 1 is aimed at those candidates who would typically score less than 40% on Level 2 papers. Level 2 will be taken by the majority of candidates. Level 3 will test the extended specification. The mental test will be common to all three levels. It will be recorded.

SCHOLARSHIP

The Common Academic Scholarship Examination (90 minutes) will be based on the extended 13+ Common Entrance syllabus.