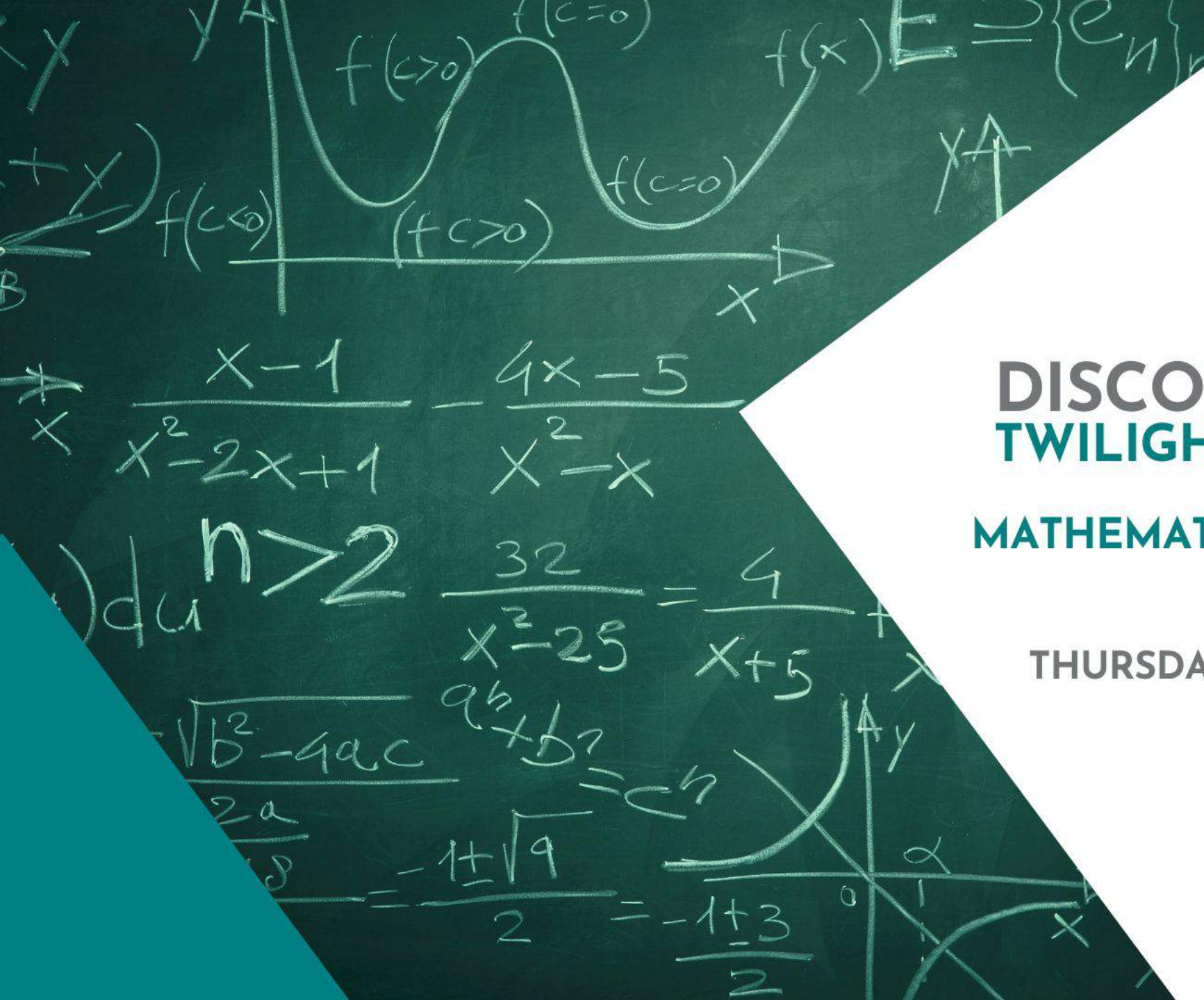


ISEB
Independent Schools Examinations Board

DISCOVER NEW CE
TWILIGHT CPD SESSIONS

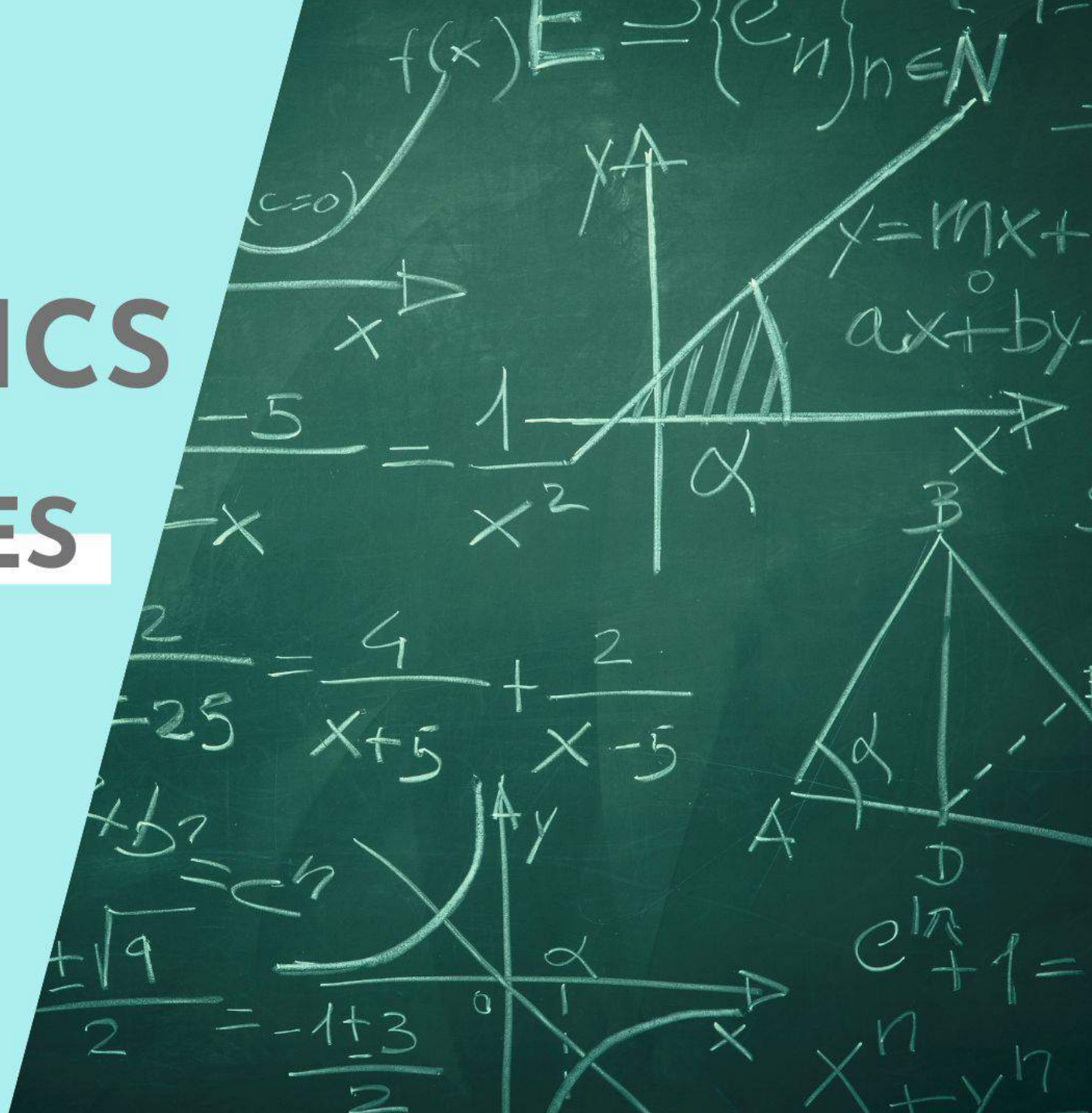
MATHEMATICS SPECIFICATIONS
PATRICK DAVIES

THURSDAY 27TH MAY | 17:00 BST

ISEB 13+ MATHEMATICS

PATRICK DAVIES

Thursday 27th May



Reasons for Change

Evolving senior school entrance requirements

Improve Mathematical foundations on which to build

Encourage more thinking and problem solving

Reduce candidates' stress

Increase candidates' sense of success and motivation

Reduce carbon footprint

What's new?

Reduction in number of topics examined

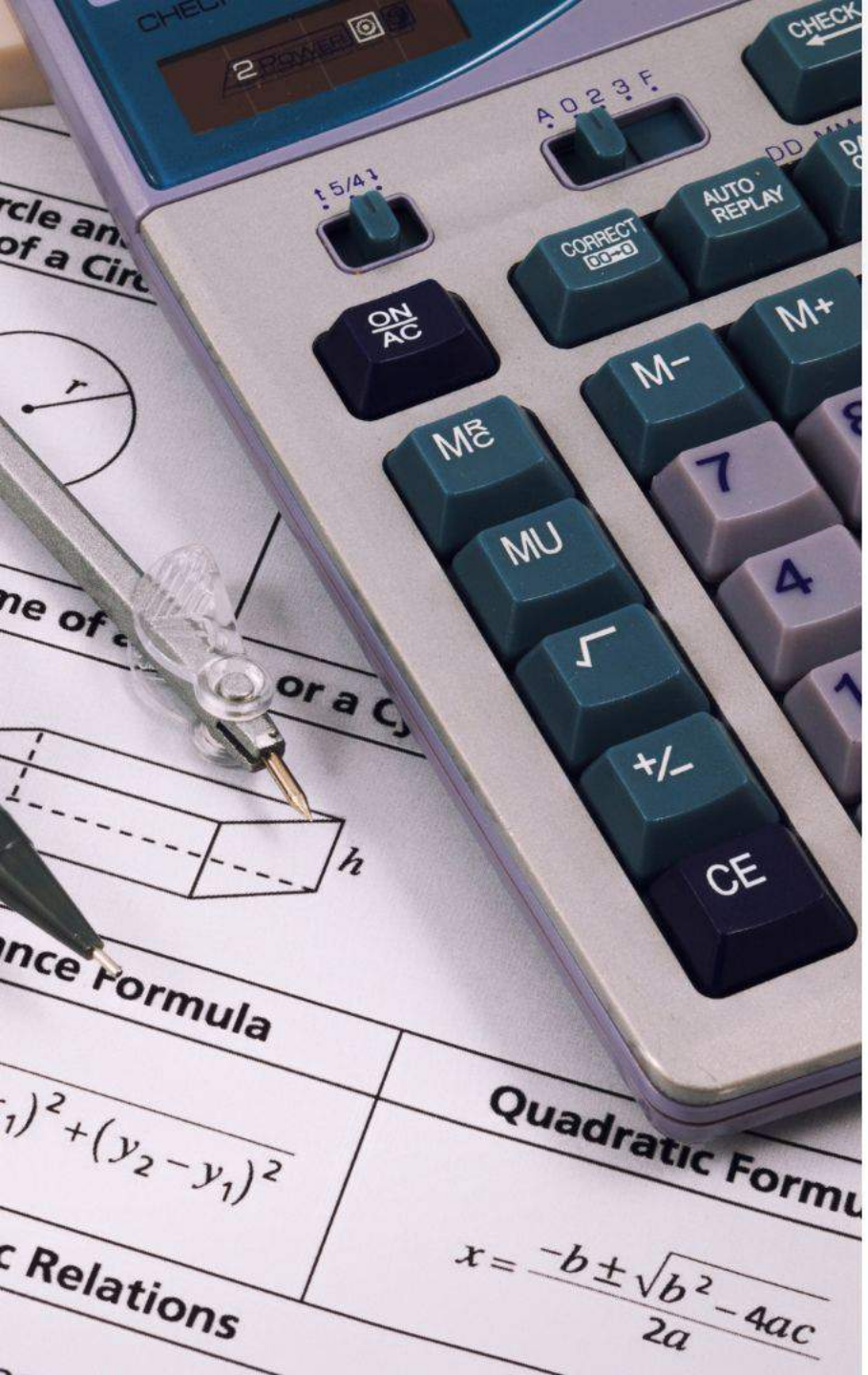
Fewer topics covered in a paper

Fewer words

More straight questions

Less scaffolding

More thought provoking, novel problems

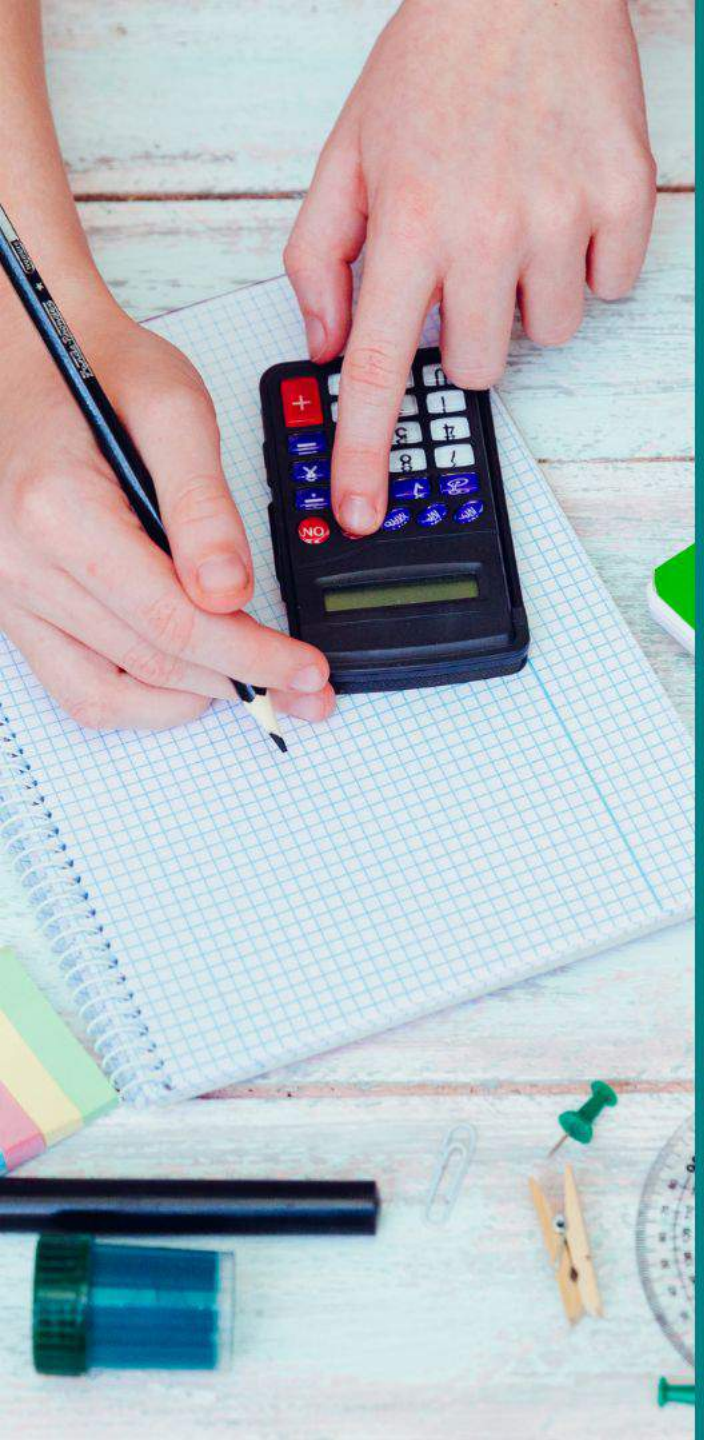


What's new?

More marks per question

Expectation of better, fuller working

Units required to be given



The New Papers

Core Papers

- Non Calculator Paper and Calculator Paper
- Taken by nearly all candidates
- Balance of basics skills, reasoning and novel problems

Additional Paper

- Calculator Paper
- Advanced topics: Pythagoras, Simultaneous Equations, etc.
- More challenging problems

The New Papers

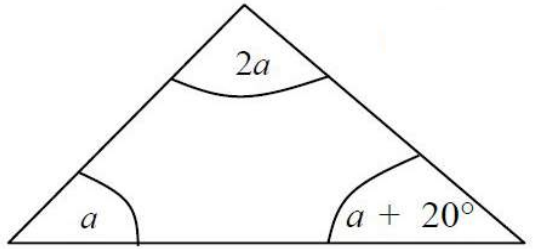
Foundation Paper

- A much more basic level
- Small number of candidates

Mental Arithmetic Paper

- Written
- More straight mental arithmetic
- More questions: greater opportunity for weaker candidates

The Mental Arithmetic Test

35.	What is the value of x when $\frac{8}{x} = 2$?	x =
36.	 Find the value of a.	a = °
37.	The length of one side of a rectangle is 7 metres. Its area is 28 square metres. What is its perimeter?	m
38.	Find the value of 423 x 19 + 423	
39.	Find the value of 58.6 ÷ 0.586	
40.	What is $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$?	

Core Non Calculator

Please read this information before the examination starts.

- This examination is 60 minutes long.
- Try as many questions as you can.
- **You must show all your working or you may receive no marks**.
- You are encouraged to cross out mistakes neatly, not erase them.
- Answers given as fractions should be in their lowest terms and written as mixed numbers where appropriate.
- **Answers should include correct units where necessary**.
- A row of dots ……. shows where to write an answer.
- Grey boxes are for your working. Extra working can be done anywhere on the paper.
- If there is no row of dots, double underline your answer with your working in the grey box.

Core Non Calculator

1. (a) Write the correct sign, **<** , **>** or **=** between each pair.

(i) **6.4** ... < ... **6.8** (ii) **8.7** ... > ... **8.14** (iii) **5.9** ... > ... **5.09**

(iv) **7.6** ... = ... **7.60** (v) **32.4** ... > ... **3.24** (vi) $\frac{3}{5}$ ... = ... $\frac{6}{10}$

(4)

(b) *Circle the correct answers.*

(i) In the number **376.91** , what does the **7** stand for?

seven tenths	**seventy**	**seven hundred**	**seven hundredths**

(ii) In the number **23.574** , what does the **7** stand for?

seven tenths	**seventy**	**seven hundred**	**seven hundredths**

(2)

2. Work out

(i) $3 + 2 \times 4$

$= 3 + 8$

$= 11$

(ii) $(7 + 3) \times 5$

$= 10 \times 5$

$= 50$

(iii) $\frac{20 - 2 \times 3 + 4}{3}$

$= \frac{20 - 6 + 4}{3}$

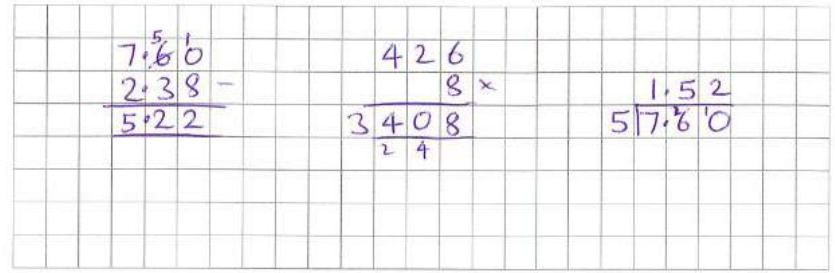
$= \frac{18}{3}$

$= 6$

(6)

3. Calculate

(i) 7.6 – 2.38 (ii) 42.6 x 0.8 (iii) 7.6 ÷ 5



5.22 34.08 1.52

(6)

4. Use the fact that **64 × 32 = 2048** to complete these:

(i) 6.4 × 320

= 2048

(ii) 2048 ÷ 32

= 64

(iii) 32 x 32

= 1024

(iv) 650 × 320

= 208000

2048
32 +
2080
1

(4)

5. Calculate

(i) $\frac{5}{7}$ of 280

$\frac{5}{7_1} \times \overset{40}{\cancel{280}}$

$= 200$

(ii) 5% of 120 kg

10% : 12 kg

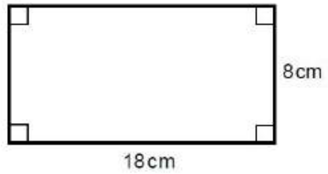
5% : 6 kg

(5)

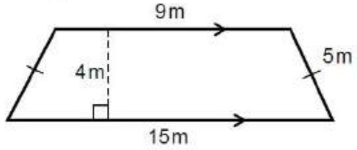
Core Non Calculator

6. Calculate the **area** of each shape.

(i)



(ii)



$A = b \times h$
$= 18cm \times 8cm$
$= \underline{\underline{144cm^2}}$

$18 \times 8 = 144$

$A = \frac{a+b}{2} \times h$
$= \frac{9+15}{2} \times 4$
$= 12 \times 4$
$= \underline{\underline{48m^2}}$

(6)

7. Change to decimals

(i) $\frac{7}{20} = \frac{35}{100}$
$= \underline{\underline{0.35}}$

(ii) $\frac{7}{8} = \underline{\underline{0.875}}$

$8 \overline{)7.000}$ = 0.875

(4)

8. (a) Write these as **products of prime factors**.

(i) 72

$= 9 \times 8$
$= \underline{\underline{3 \times 3 \times 2 \times 2 \times 2}}$

(ii) 180

$= 18 \times 10$
$= 6 \times 3 \times 10$
$= \underline{\underline{2 \times 3 \times 3 \times 2 \times 5}}$

(4)

(b) Find the smallest number you need to multiply 180 by to make a square number.

$(2 \times 3 \times 5) \times (2 \times 3 \times \underline{5})$

Answer: ....5........ (1)

9. Calculate

(i) $\frac{7}{8} - \frac{1}{8}$

$= \frac{6}{8}$
$= \underline{\underline{\frac{3}{4}}}$

(ii) $\frac{5}{9} + \frac{1}{6}$

$= \frac{10}{18} + \frac{3}{18}$
$= \underline{\underline{\frac{13}{18}}}$

(iii) $\frac{9}{20} \div \frac{3}{14}$

$= \frac{9}{20} \times \frac{14}{3}$
$= \frac{21}{10}$
$= \underline{\underline{2\frac{1}{10}}}$

(6)

Core Non Calculator

10. A pony eats $\frac{4}{9}$ of a bale of hay each day.

How many bales of hay will it eat in 45 days?

no. of bales $= \frac{4}{9_1} \times 45^5$

$= 20$

(3)

11. If $a = 5$, $b = -4$ and $c = 3$, find the value of

(i) $3a + 2b$

$= 15 - 8$

$= 7$

(ii) $b^2 - a$

$= 16 - 5$

$= 11$

(iii) $\frac{4a - b}{2}$

$= \frac{20 - -4}{2}$

$= \frac{24}{2} = 12$

(iv) $(2a)^2 - 2c^2$

$= 10^2 - 2 \times 9$

$= 100 - 18$

$= 88$

12. Solve

(i) $5z - 2 = 33$

$5z = 35$

$z = 7$

(ii) $\frac{3m + 1}{3} = 4$

$3m + 1 = 12$

$3m = 11$

$m = \frac{11}{3}$

$= 3\frac{2}{3}$

(4)

13. **Estimate** the value of

(i) 3.92×12.17

$\approx 4 \times 12$

$= 48$

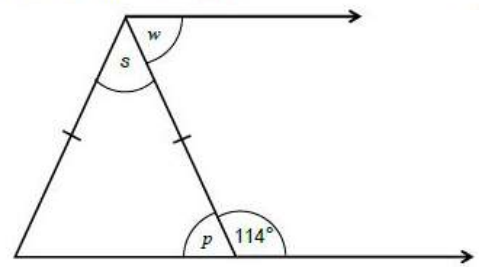
(ii) $\frac{179.31 - 31.3}{29.94}$

$\approx \frac{180 - 30}{30}$

$= \frac{150}{30} = 5$

(4)

14. Calculate the size of each lettered angle.



$p = 180° - 114°$

$= 66°$

$w = 66°$

$s = 180° - 2 \times 66°$

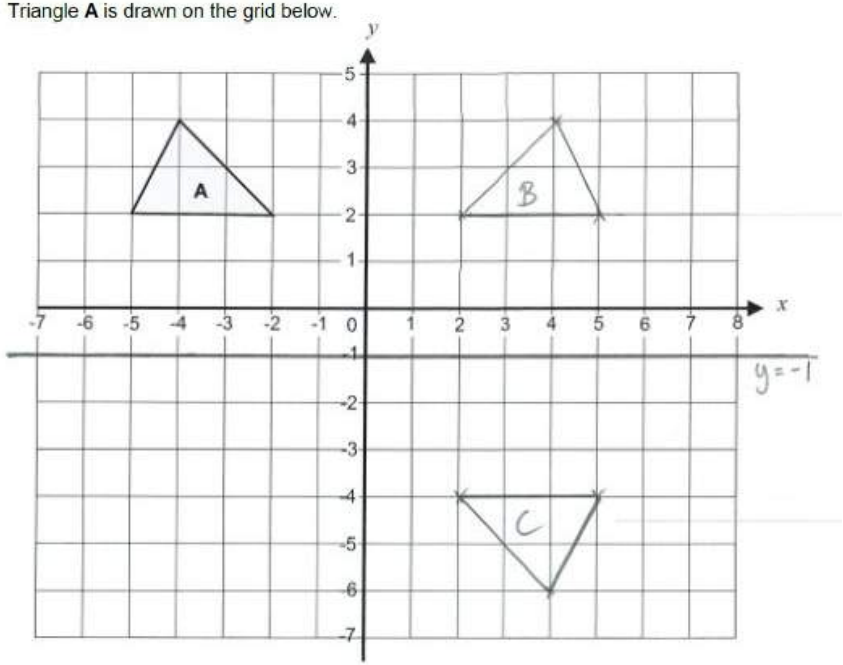
$= 180° - 132°$

$= 48°$

(5)

Core Non Calculator

15. Triangle **A** is drawn on the grid below.



(i) **Reflect** triangle **A** in the y-axis.
Label the new triangle **B**. (2)

(ii) **Reflect** triangle **B** in the line $y = -1$
Label the new triangle **C**. (2)

(iii) A **rotation** can map triangle **C** onto triangle **A**.
Plot the centre of the rotation and label it *P*.
Write the coordinates of *P* and the angle of rotation.

coordinates: (0, -1)

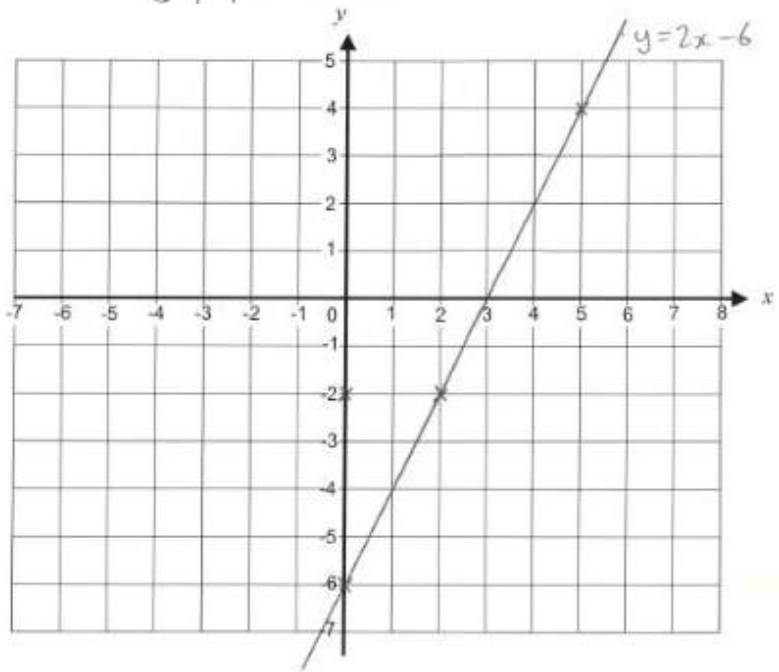
angle: 180°

(2)

16. (a) On the centimetre-square grid below, draw and label the line

$y = 2x - 6$

x	5	2	0
y	4	-2	-6



(b) Work out the area of the triangle formed by your line, the x-axis and the y-axis.

$A = \frac{b \times h}{2}$

$= \frac{3 \times 6}{2}$

$= 9\text{cm}^2$

(2)

Core Non Calculator

17. Look at the sequence of patterns made from matches.

pattern 1 pattern 2 pattern 3

(a) How many matches would you need to make **pattern 6**?

1 2 3 4 5 6

5 10 16 23 31 40

+5 +6 +7 +8 +9

40

(b) Write down the size of the **area** inside **pattern 3**

Answer: 15 (1)

(c) Which pattern in the sequence would have an area of 143 units^2?

Area $= (n+1)^2 - 1$

$(11+1)^2 - 1$

$= 144 - 1$

$= 143$

$n = 11$

(2)

20. (a) Calculate

(i) 1^3 (ii) $1^3 + 2^3$ (iii) $1^3 + 2^3 + 3^3$ (iv) $1^3 + 2^3 + 3^3 + 4^3$

(b) What do you notice about the results?

(c) Use your idea to calculate $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$

(a) $1^3 = 1$ $1^3 + 2^3 = 1 + 8 = 9$ $1^3 + 2^3 + 3^3 = 9 + 27 = 36$ $1^3 + 2^3 + 3^3 + 4^3 = 36 + 64 = 100$

(b) They are square nos

1^2, 3^2, 6^2, 10^2

(c) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3$

$= 21^2$

$= 441$

1, 3, 6, 10, 15, 21

21
21
21
420
441

(5)

(Total marks: 100)

Core Calculator

1 (a) Use your calculator to work out the value of

$$1.8 + 4.13^2$$

Answer: 24.1729 (2)

(b) Write your answer to part (a) correct to 2 decimal places.

Answer: 24.17 (1)

2 (a) Write the ratio **24 : 18** in its simplest form.

4 : 3

Answer: 4:3 (2)

(b) The ratio of boys to girls at a school is **7 : 3**

If there are 120 children altogether, how many girls are there?

B : G

7 : 3 (10) ×12

84 : 36 (120)

36 girls

(3)

3 Write the next three numbers in each sequence.

(i) **7 , 11 , 15 , 19** , 23, 27, 31

(2)

(ii) **16 , 8 , 4** , 2, 1, $\frac{1}{2}$

(2)

4 (a) Jenson buys a car for **£127 500**

He sells it to Lewis for 12% more.

How much does Lewis pay?

new price = 1.12 × 127500

= £142800

(3)

(b) In one year, Greta grows from **140 cm** to **161 cm**.

Calculate her **percentage increase** in height?

% increase = $\frac{21}{140}$

= 15%

Core Calculator

5. Complete these

(i) 365.7 + 527.5 = 893.2

(ii) 15.9 x 7.3 = 116.07

(iii) 46.8 ÷ 5.2 = 9

(iv) 24.68 − (55.32) = 80

(4)

6. (a) Simplify

(i) $s + s$

= 2s

(ii) $t \times t$

= t^2

(2)

(iii) $8a + 3 - 2a + 5$

= 6a + 8

(iv) $7c^2 \times 3c$

= $21c^3$

(4)

(b) Multiply out the brackets and simplify

(i) $5e + 2(3e - 4)$

= 5e + 6e − 8

= 11e − 8

(ii) $10 - 3(3z - 2)$

= 10 − 9z + 6

= 16 − 9z

(4)

7. Factorise fully

(i) $18x - 12$

$= 6(3x - 2)$

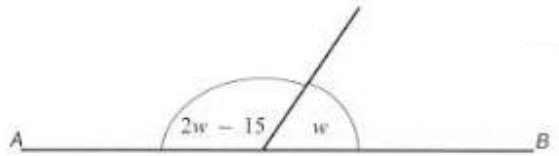
(ii) $25y + 5$

$= 5(5y + 1)$

(3)

8. In the diagram, AB is a straight line and the angles are marked in degrees.

Work out the size of angle w



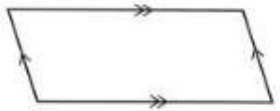
$2w - 15 + w = 180°$

$3w = 195$

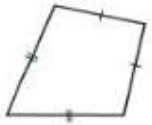
$w = 65°$

Core Calculator

9. Write the name of each shape.



Answer: parallelogram

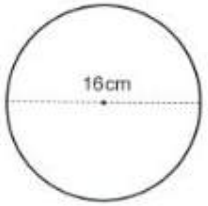


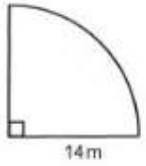
Answer: kite

(2)

10. *For this question, give answers correct to the nearest whole number.*

(a) (i) Calculate the **circumference** of this circle. (ii) Calculate the **area** of this quadrant.





$C = \pi D$

$= \pi \times 16$

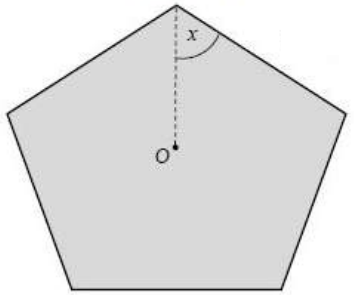
$= 50.3\text{ cm}$ (1 d.p)

$A = \frac{\pi r^2}{4}$

$= \frac{\pi \times 14^2}{4}$

$= 153.9\text{ m}^2$ (1 d.p)

12. (a) Here is a regular pentagon with centre O.



Calculate the size of angle x

$a = \frac{360^\circ}{5}$

$= 72^\circ$

$b = 180^\circ - 72^\circ$

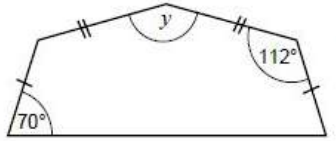
$= 108^\circ$

$x = \frac{108^\circ}{2} = 54^\circ$

(5)

(b) Here is another pentagon.

Calculate the size of angle y



sum of int. $\angle$s

$= 108^\circ \times 5$

$= 540^\circ$

$y = 540^\circ - (140^\circ + 224^\circ)$

$= 540^\circ - 364^\circ$

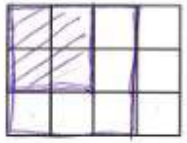
$= 176^\circ$

(3)

Core Calculator

16. Here is a rectangle divided up into squares.

How many squares are here altogether?



3by3 : 2
2by2 : 6
1by1 : 12

Total = 2 + 6 + 12
= 20

17. **Emily** has twice as many marbles as **James**.
Edward has 3 more marbles than **James**.
The three children have 23 marbles altogether.

How many marbles does James have?

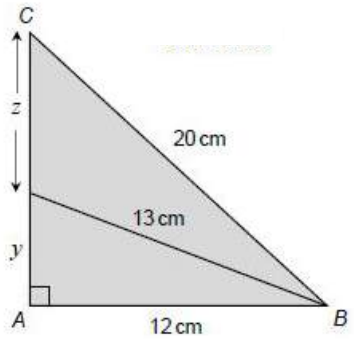
James : x, Emily: $2x$, Ed: $x+3$

$x + 2x + x + 3 = 23$
$4x + 3 = 23$
$4x = 20$
$x = 5$

(3)

The Additional Paper

1. (a) Calculate the lengths marked y and z in the triangle ABC below.



$y = \sqrt{13^2 - 12^2}$
$= \sqrt{25}$
$= 5\text{cm}$

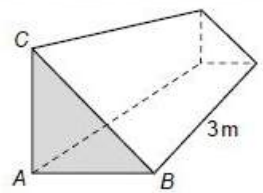
$AC = \sqrt{20^2 - 12^2}$
$= \sqrt{256}$
$= 16$

$z = 16 - 5$
$= 11\text{cm}$

(6)

(b) ABC forms the perpendicular cross-section of a prism, 3 metres long, made from lead.

Calculate the **volume** of the prism in cubic centimetres.



$V = \frac{b \times h}{2} \times l$
$= \frac{12 \times 16}{2} \times 300$
$= 28800\text{cm}^3$

(4)

(c) 1 cubic metre of lead has a mass of 11342 kilograms.

Calculate the **mass** of the wedge.

$V = \frac{28800}{1000000} = 0.0288\text{m}^3$

$\text{mass} = 11342 \times 0.0288$
$= 327\text{kg}$ (3sf)

(3)

3. Solve the equation $\frac{x+1}{3} = 3 - \frac{x-1}{4}$

$4x + 4 = 36 - 9x + 3$
$13x = 35$
$x = \frac{35}{13}$
$= 2\frac{9}{13}$

4. When 2 adults and 3 children go to the zoo it costs £48
However, it costs £76 for 5 adults and 2 children go to the zoo.

Work out the price of an **adult's ticket** and the price of a **child's ticket**.

$2a + 3c = 48$ ①
$5a + 2c = 76$ ②

① × 2 ⇒ $4a + 6c = 96$ ③
② × 3 ⇒ $15a + 6c = 228$ ④

④ − ③ ⇒ $11a = 132$
$a = 12$

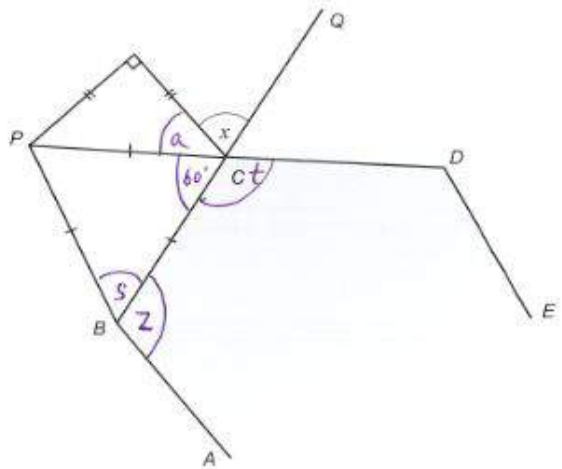
subst $a \to$ ① ⇒ $24 + 3c = 48$
$3c = 24$
$c = 8$

child: £8
adult: £12

(6)

The Additional Paper

6. In the diagram, *PCD* and *BCQ* are straight lines.



$a = \frac{180° - 90°}{2}$

$= 45°$

(a) Calculate the size of angle x

$x = 180° - (45° + 60°)$

$= 180° - 105°$

$= 75°$

(4)

ABCDE forms part of a regular polygon.

(b) Calculate the number of sides of the complete polygon.

$n = \frac{360°}{60°}$

$= 6$

(2)

(c) What can you say about *PBA* and why?

PCD is straight line $\Rightarrow t = 180° - 60°$

$z = 120°$ (int. $\angle$s are equal) $= 120°$

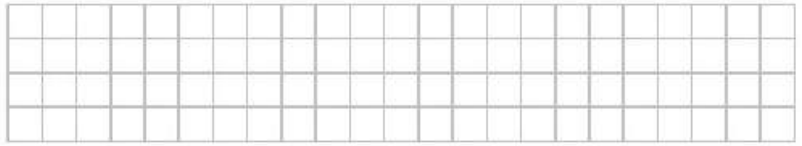
$S = 60°$

$z + S = 180° \Rightarrow$ PBA is straight line

Foundation

(b) Calculate

(i) 385 ÷ 5 (ii) 74 x 3



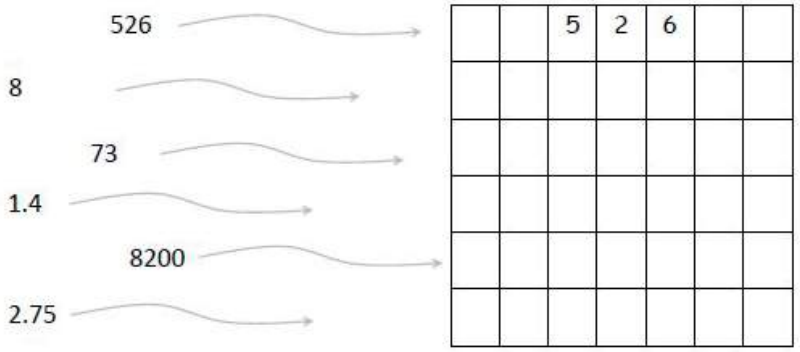
[4]

2. Circle the bigger number in each pair.

(a) 68 86 (b) 54 540 (c) 4.16 4.8

[2]

3. Write these numbers in the correct columns.



[3]

5. True or False? (Write T or F)

(a) 12 – 2 = 2 – 12 ..... (b) 15 x 7 = 7 x 15 .....

(c) 73 ÷ 4 = 4 ÷ 73 ..... (d) 9^2 = 9 x 2 .....

[2]

6. Circle the correct operation for each question.

(a) What is the sum of 8 and 3?

8 x 3	3 – 8	8 – 3	8 ÷ 3	8 + 3

[1]

(b) Tom buys 12 bags of nuts.

He buys 480 nuts altogether.

How many nuts are in each bag?

12 x 480	12 ÷ 480	480 x 12	480 ÷ 12	480 – 12

[1]

(c) Sarah eats 6 chocolates a day.

How many chocolates will she eat in 150 days?

150 ÷ 6	6 ÷ 150	150 x 6	150 – 6	6 – 150

[1]

(d) How much bigger is 74 than 28?

74 + 28	28 – 74	28 + 74	74 x 28	74 – 28

[1]

Foundation

7. Find $\frac{2}{5}$ of 15

[2]

8. Calculate

(a) 53.8 + 2.47 (b) 83 – 8.3

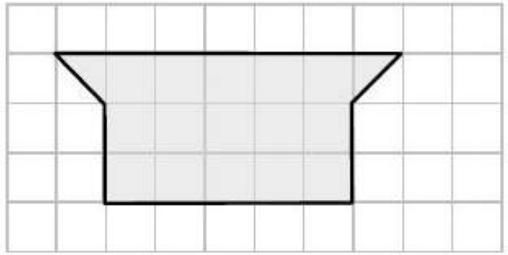
[4]

9. (a) Which is bigger? $\frac{3}{4}$ or $\frac{3}{8}$

Circle the correct answer. $\frac{3}{4}$ $\frac{3}{8}$ they are the same size

[1]

11. Write down the **area** of the shape drawn on this centimetre grid.



Area = ...

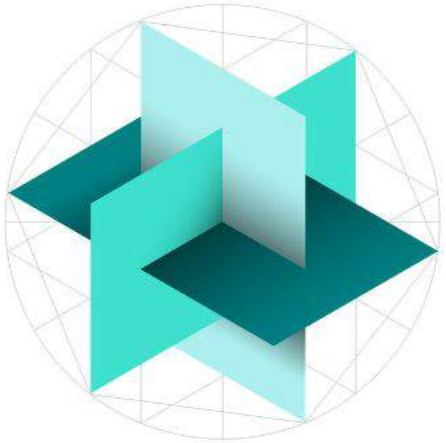
[2]

12. Change $\frac{4}{5}$ to a decimal

[2]

13. Find the **area** of this rectangle.

[3]



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