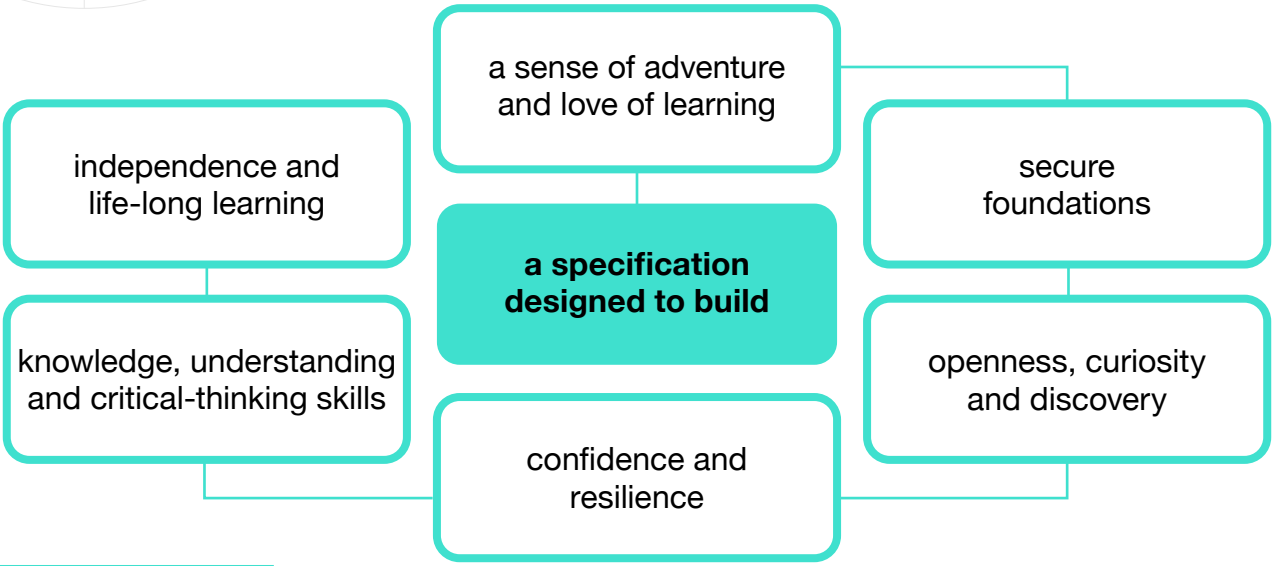
ISEB
Independent Schools Examinations Board

MATHEMATICS

CE AT 11+ & 13+
COMMON ACADEMIC SCHOLARSHIP AT 13+

Specification
For teaching from September 2021 onwards
For examinations from November 2022 onwards



ISEB CORE AIMS

Pupils who have pursued a course of study based on CE specifications and assessments will:

- be equipped not only for the next stage of their education, but for life-long learning based on a secure foundation of subject knowledge, concepts and skills and be able to apply what they know to new situations
- be enthusiastic learners who are open to new ideas and experiences, curious, questioning and keen to experiment.

They will:

- enjoy reading and be able to articulate clearly orally and in writing
- have the confidence to think, weigh up evidence and make up their own minds, and the resilience to learn from their mistakes
- have the skills to work independently and collaboratively
- understand how subjects connect with each other
- demonstrate cultural and environmental awareness and empathy, developing an understanding of their place in the world.

IMPORTANT INFORMATION | DISCLAIMER

Specifications are updated over time. Whilst every effort is made to check all documents, there may be contradictions between published resources and the specification, therefore please use the information on the latest specification at all times.

When we make changes to the specifications:

- we will indicate the change clearly in th specification
- there will be a new version number indicated
- a summary of changes will be published as a seperate document

If you do notice a discrepancy between the specification and a resource please contact us at: common-entrance@iseb.co.uk

INTRODUCTION

N.B. The content of this specification is a revision of the syllabus released in 2014 for 13+ and CASE. The content of this specification for 11+ is unchanged.

The CE mathematics specification is designed to instil in pupils the importance of the subject and its associated skills. It should motivate, inspire, encourage and reward pupils for their progress and achievements in mathematics. The specification should encourage a breadth of experience in the development of pupils' mathematical skills. It should encourage the development of investigative thinking, reasoning and the application of mathematical knowledge to unfamiliar situations. It should provide pupils with a solid foundation for their future learning and give them a degree of confidence in themselves and the subject.

AIMS

A course leading to the 11+ and 13+ examinations should:

- enable pupils to become fluent with fundamental mathematical concepts
- encourage the development of investigative thinking, problem solving and reasoning skills
- develop pupils' analytical skills, enabling them to select appropriate methods
- encourage pupils to work logically and express mathematical ideas clearly, correctly and succinctly
- instil confidence and resilience through an appreciation of the value of learning from mistakes
- promote the idea that everyone can be successful in mathematics
- motivate and reward pupils by enabling them to recognise and take satisfaction from their progress
- instil in pupils the importance of the subject and its associated skills, including the applications of mathematics in other disciplines
- enable pupils to recognise the beauty of mathematics through an appreciation of the simplicity and elegance with which mathematics expresses profound and interconnected ideas
- provide a solid foundation for future progress.

ASSESSMENT OBJECTIVES

11+

Candidates should be familiar with the skills and knowledge of the National Curriculum key stage 2 programmes of study. The principal focus of mathematics teaching at 11+ is:

- to ensure that pupils extend their understanding of the number system and place value to include larger integers
- to develop the connections which pupils make between multiplication and division with fractions, decimals, percentages and ratio
- to develop pupils' ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation
- to introduce pupils to the language of algebra as a means for solving a variety of problems
- to consolidate and extend knowledge developed in number in geometry and measure; to ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them
- to ensure that pupils are fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages
- to ensure that pupils read, spell and pronounce mathematical vocabulary correctly.

13+

During key stage 3, candidates build on the skills which they developed in key stage 2 and increasingly make connections between different aspects of mathematics. Candidates should build on key stage 2 and make connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also be able to apply their mathematical knowledge in science, geography, computing and other subjects.

Candidates should be familiar with the skills and knowledge of the National Curriculum key stage 3 programmes of study ***as specified in the summary*** (page 30).

ASSESSMENTS

11+ MATHEMATICS

CE at 11+	Marks	
One paper	100	60 minutes

Topics are centred on the National Curriculum for key stage 2 with particular reference to the Year 5 and 6 programmes of study; because of the timing of the 11+ examinations in November and January, not all statements will be examined, but it is recommended that those topics not included in the examination specification should still be taught in Year 6.

Candidates will be required to work one paper of 60 minutes' duration. All candidates should answer as many questions as possible. Whilst candidates should be familiar with the use of calculators, calculating aids may not be used in the examination.

13+ MATHEMATICS

The expectation is that all candidates will take the Core Papers. An Additional Paper is available to offer challenge to the more able.

All candidates will be required to take two papers, one non-calculator and one calculator, each of 60 minutes' duration, and a short mental arithmetic test (10 minutes).

Candidates who show capability will **also** have the option of taking the additional paper, of 60 minutes duration. Calculators may be used for this paper.

However, the Foundation Papers are available for support in individual circumstances. These papers may be used for assessment at CE 13+ by agreement with senior schools, or as practice and stepping-stones towards CE.

CE at 13+ Foundation Papers (for support, if required)		**Marks**	
Foundation	Non-calculator	100	60 minutes
Foundation	Calculator	100	60 minutes
Mental Arithmetic Test*		40	10 minutes

CE at 13+ Core Papers (suggested for all candidates)		**Marks**	
Core	Non-calculator	100	60 minutes
Core	Calculator	100	60 minutes
Mental Arithmetic Test*		40	10 minutes

CE at 13+ Additional Paper (optional)	**Marks**	
Additional Paper	100	60 minutes

Common Academic Scholarship	**Marks**	
One paper	100	90 minutes

The Common Academic Scholarship Examination (90 minutes) will be based on the 13+ CE specification ***for the Additional Paper.***

** the same mental arithmetic test is taken by both Foundation and Core candidates*

11+ SUBJECT CONTENT

In the specification below:

- the complete key stage 2 programmes of study for Years 5 and 6 are shown in the left-hand column
- *elements of the programme shown in italics will not be examined* at CE, but it is recommended that they still be taught before the end of Year 6; including non-statutory work
- further guidance and clarification about what will be examined at 11+ is given in the right-hand column
- for more detailed information, preparatory schools are advised to look at the *National Curriculum in England: Mathematics Programmes of Study* document, https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study published in September 2013 and updated in July 2014, including non-statutory notes and guidance.

11+ MATHS

National Curriculum descriptors	11+ examination guidance
NUMBER	
Pupils should be taught to:	
number and place value – Year 5	
read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit	
count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000	
interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero	
round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000	
solve number and practical problems which involve all of the above	
read Roman numerals to 1000 (M) and recognise years written in Roman numerals	
number and place value – Year 6	
read, write, order and compare numbers to at least 10 000 000 and determine the value of each digit	
round any whole number to a required degree of accuracy	
use negative numbers in context, and calculate intervals across zero	
solve number and practical problems which involve all of the above	include multistep problems, using the four operations and any relevant skills from those listed

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
addition, subtraction, multiplication and division – Year 5	
add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	
add and subtract numbers mentally with increasingly large numbers	
use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	
identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers	
know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	
establish whether a number up to 100 is prime and recall prime numbers up to 19	
multiply numbers up to 4 digits by a one- or two-digit whole number using a formal written method, including long multiplication for two-digit numbers	
multiply and divide numbers mentally drawing upon known facts	include mental recall of multiplication and division facts up to 12 x 12
divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	
multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	
recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	
solve problems involving addition, subtraction, multiplication and division including using their knowledge of factors and multiples, squares and cubes	
solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	
solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
addition, subtraction, multiplication and division – Year 6	
multiply multidigit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	
divide numbers up to 4 digits by a two-digit number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	*only division by numbers up to 12 or multiples of 10 will be examined, but teaching of division by other two-digit numbers in Year 6 is advised*
divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context	*only division by numbers up to 12 or multiples of 10 will be examined, but teaching of division by other two-digit numbers in Year 6 is advised*
perform mental calculations, including with mixed operations and large numbers	
identify common factors, common multiples and prime numbers	
use their knowledge of the order of operations to carry out calculations involving the four operations	
solve addition and subtraction multistep problems in contexts, deciding which operations and methods to use and why	
solve problems involving addition, subtraction, multiplication and division	
use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy	

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
fractions (including decimals and percentages) – Year 5	
compare and order fractions whose denominators are all multiples of the same number	
identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	
recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} + \frac{6}{5} = \frac{12}{5}$]	
add and subtract fractions with the same denominator and denominators which are multiples of the same number	
multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	
read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$]	decimals up to three decimal places
recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	
round decimals with two decimal places to the nearest whole number and to one decimal place	
read, write, order and compare numbers with up to three decimal places	
solve problems involving numbers up to three decimal places	
recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal	
solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25	solve number, measure, money and practical problems involving listed fraction, decimal and percentage skills

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
fractions (including decimals and percentages) – Year 6	
use common factors to simplify fractions; use common multiples to express fractions in the same denomination	
compare and order fractions, including fractions > 1	*examination limited to fractions whose denominations are multiples of the same number*, e.g. $\frac{5}{6}$ and $\frac{11}{12}$
add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	only fractions whose denominators are multiples of the same number will be examined and not mixed numbers, but teaching in Year 6 is advised
multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]	
divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]	*this will not be examined, but teaching in Year 6 is advised*
associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]	*this will not be examined, but teaching in Year 6 is advised*
identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places	*this will not be examined, but teaching in Year 6 is advised*
multiply one-digit numbers with up to two decimal places by whole numbers	
use written division methods in cases where the answer has up to two decimal places	*this will not be examined, but teaching in Year 6 is advised*
solve problems which require answers to be rounded to specified degrees of accuracy	*this will not be examined, but teaching in Year 6 is advised*
recall and use equivalences between simple fractions, decimals and percentages, including in different contexts	limited to halves, quarters, fifths, tenths and hundredths

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
RATIO and PROPORTION	
Pupils should be taught to:	
Year 6 only	
solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts	
solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison	examination of percentages limited to multiples of 10, or 25% or 75%; but teaching in Year 6 of other percentages is advised
solve problems involving similar shapes where the scale factor is known or can be found	
solve problems involving unequal sharing and grouping using knowledge of fractions and multiples	
enumerate possibilities of combinations of two variables	
ALGEBRA	
Pupils should be taught to:	
Year 6 only	
use simple formulae	will involve simple words and letters NB: the treatment of algebra will be largely informal with the emphasis upon understanding that letters can represent unknowns and variables
generate and describe linear number sequences	will include finding term-to-term rules
express missing number problems algebraically	
find pairs of numbers that satisfy an equation with two unknowns	
enumerate possibilities of combinations of two variables	
use simple formulae	will involve simple words and letters NB: the treatment of algebra will be largely informal with the emphasis upon understanding that letters can represent unknowns and variables
generate and describe linear number sequences	will include finding term-to-term rules
express missing number problems algebraically	
find pairs of numbers that satisfy an equation with two unknowns	
enumerate possibilities of combinations of two variables	

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
MEASUREMENT	
Pupils should be taught to:	
Year 5	
convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)	
understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints	
measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres	
calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	
estimate volume [for example, using 1-cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]	
solve problems involving converting between units of time	including solving problems involving time and timetables
use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling	
Year 6	
solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate	*this will not be examined, but teaching in Year 6 is advised*
use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places	
convert between miles and kilometres	*this will not be examined, but teaching in Year 6 is advised*
recognise that shapes with the same areas can have different perimeters and vice versa	
recognise when it is possible to use formulae for area and volume of shapes	*candidates should recognise when it is possible to use formulae to calculate the areas of right-angled triangles, including standard units of cm^2 and m^2, otherwise this will not be examined, but teaching in Year 6 is advised*
calculate the area of parallelograms and (non right-angled) triangles	only area of right-angled triangles *will* be examined, but teaching of areas generally in Year 6 is advised
calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]	find the volume of shapes by counting cubes and know the unit cm^3 *will* be examined

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
GEOMETRY – properties of shape	
Pupils should be taught to:	
Year 5	
identify 3-D shapes, including cubes and other cuboids, from 2-D representations	
know angles are measured in degrees; estimate and compare acute, obtuse and reflex angles	
draw given angles, and measure them in degrees (°)	
identify: > angles at a point and one whole turn (total 360°) > angles at a point on a straight line and a half turn (total 180°) > other multiples of 90°	
use the properties of rectangles to deduce related facts and find missing lengths and angles	
distinguish between regular and irregular polygons based on reasoning about equal sides and angles	
Year 6	
draw 2-D shapes using given dimensions and angles	
recognise, describe and build simple 3-D shapes, including making nets	*this will not be examined, but teaching in Year 6 is advised*
compare and classify geometric shapes based on their properties and sizes and *find unknown angles in any triangles, quadrilaterals, and regular polygons*	*only finding angles in triangles and rectangles will be examined, but teaching in Year 6 of finding other unknown angles is advised*
illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius	*this will not be examined, but teaching in Year 6 is advised*
recognise angles where they meet at a point, are on a straight line, or are vertically opposite angles and angles in a triangle, and find missing angles	

11+ MATHS (continued)

National Curriculum descriptors	11+ examination guidance
GEOMETRY – position and direction	
Pupils should be taught to:	
Year 5	
identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed	recognise that shapes with the same areas can have different perimeters and vice versa draw 2D shapes using given dimensions and angles
Year 6	
describe positions on the full coordinate grid (all four quadrants)	only describing and plotting positions on a 2D grid as coordinates in the first quadrant *will* be examined, but teaching in Year 6 is advised
draw and translate simple shapes on the coordinate plane, and *reflect them in the axes*	*reflecting in coordinate axes will not be examined, but teaching in Year 6 is advised*
STATISTICS	
Pupils should be taught to:	
Year 5	
solve comparison, sum and difference problems using information presented in a line graph	
complete, read and interpret information in tables, including timetables	to include information presented in bar charts, pictograms, tables, line graphs and time graphs the collection and recording of discrete data including in grouped frequency tables, to include tallying
Year 6	
interpret (*and construct*) pie charts and line graphs and use these to solve problems	candidates will be expected to interpret pie charts, including connecting their work on angles, fractions, and percentages; *the construction of pie charts will not be examined but teaching in Year 6 is advised*
calculate and interpret the mean as an average	data limited to discrete values in lists as frequency tables (but not grouped frequency tables)

13+ SUBJECT CONTENT

In the specification below:

- a brief outline of topics for the 13+ Core and Foundation Papers is provided in the first table
- an outline of further topics for the Additional Paper is provided in the second table
- further guidance and examples of what will be examined at 13+ is given in the centre and right-hand columns
- National Curriculum topics are provided in the final table to indicate which elements are examined at Core/ Foundation and/or Additional level
- for more detailed information, prep schools are advised to look at the *National Curriculum in England: Mathematics Programmes of Study* document, https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study published in September 2013 and updated in July 2014, including non-statutory notes and guidance.

CORE LEVEL

Pupils should have knowledge and understanding of:		Further guidance	Examples *(not an exhaustive list)*
NC	**NC**	**Number**	
Place Value	1	Whole number and decimal place value Value of digits Placing numbers on number lines x and ÷10, etc.	
Ordering <, >, =	*2*	Ordering integers, decimals, fractions and percentages, including negatives and comparing using <, >, = signs	$5.7 > 5.14$ $\frac{3}{5} = \frac{6}{10}$ $3.08, 3.16, 3.8, 3.9, 31.6$
Prime numbers **Factors** **Multiples** **Common factors** **Common multiples** **Highest Common Factor** **Lowest Common Multiple** **Product of prime factors**	3	Identify prime numbers List all factors of a number List multiples HCF and LCM tested implicitly Indices may be required for product of prime factors	$120 = 12 \times 10 = \underline{\underline{3 \times 2 \times 2 \times 2 \times 5}}$ $105 = \underline{\underline{5 \times 3 \times 7}}$ What is the largest number that divides exactly into 120 and 105?
4 operations including written formal methods for integers, decimals and fractions	4	Arithmetic with fractions will be restricted to vulgar fractions but may require answers as mixed numbers Related word problem questions Fluency in times tables up to 12 x 12 expected	$12.8 + 5.72$ $58 - 5.8$ 1.7×0.6 218×43 $87 \div 0.3$ $\frac{8}{15} \times \frac{9}{20}$ $\frac{5}{8} + \frac{7}{12}$ $\frac{5}{9} - \frac{2}{9}$ $12 \div \frac{2}{3}$

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
		Number	
Mixed operations	5	Including squares, cubes, square roots and use of brackets	$15 - 3 \times 4 + 1$ $= 15 - 12 + 1$ $= \underline{\underline{4}}$ $2 \times 3^2 - (3 + 1)$ $= 2 \times 9 - 4$ $= \underline{\underline{14}}$ $4 - 12 \div 2$ $= 4 - 6$ $= \underline{\underline{-2}}$
Inverse operations	6	Understand the relationships between operations and their inverses	
Powers and Roots	7	Including powers of negative numbers	$\sqrt{81} + (-2)^3 - \sqrt[3]{27}$
Conversion between Fractions, decimals and percentages	9	Answers may have recurring decimals	$0.4 = \frac{4}{10}$ $= \underline{\underline{\frac{2}{5}}}$ $\quad 0.35 = \frac{35}{100}$ $= \underline{\underline{\frac{7}{20}}}$ Convert to decimals: $\frac{7}{20}$, $\frac{3}{8}$, $\frac{5}{6}$
Expressing one quantity as a fraction of another	10		Tim has £7 and spends 42p. What fraction has he spent?
Finding percentages of quantities	10	Non-calculator methods for simple percentages, such as 5% or 30% Calculator methods for harder examples	$10\%: £3.20$ $20\%: \underline{\underline{£6.40}}$ increase $= 0.23 \times £4500$ $= \underline{\underline{£1035}}$
Expressing one quantity as a percentage of another	10		Write 600g as a percentage of 2kg

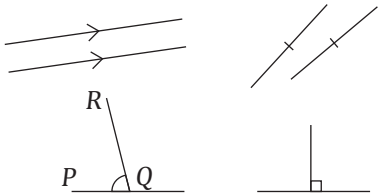
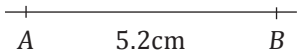
Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
		Algebra	
Changing a quantity by a percentage	10	Usually using a calculator Calculating profit and loss	A TV cost £250 but is increased in price by 17% Find the new price.
Finding a percentage change	10	Usually using a calculator	A shirt is bought for £56 and is sold for £44.80 Find the percentage loss.
Rounding numbers	13	Including to nearest 100, 10, whole number and decimal places Answers will be expected to be rounded sensibly where appropriate (Significant figures not required)	
Estimation	14	Rounding numbers sensibly to aid estimation Associated word problems	$\frac{348.2}{70.63} \approx \frac{350}{70}$ $= 5$ Estimate the cost of 41 books at £19.97 each.
Ratio	38 39 40 41	Expressing in simplest form Multiplying up to a given total or difference	$8 : 12$ $2 : 3$ $3 : 5$ $21 : 35$ 35 cats
Use of calculator	15	Including use of ( ) $\sqrt{\ }$ x^2 x^y π A simple scientific calculator is recommended. Calculators with more advanced features, such as graph plotting and solving equations, are not permitted.	

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
Algebra			
Simplification	17 20	× and ÷ Collecting like terms Multiplying out brackets Factorising by numerical factor	$3a^2 \times 2a$ $(3a)^2 + 2a^2$ $\frac{9a - a}{2}$ $\frac{15x}{3x}$ $5a - 2b - a + 4b$ $3a^2 + 2a^2$ $10 - 3(2a + 3)$ Factorise: $18a + 12$
Substitution	18	Substituting, including negatives and simple fractions, into expressions and given formulae	$a = 3,\ b = -5,\ c = \frac{1}{2}$ Find the value of: $2a^2 - 3b$ $\frac{\sqrt{10a + b}}{2}$ $\frac{a}{c}$
Forming expressions	19		Jane has x stars. Peter has 3 fewer than Jane. Sonia has twice as many as Peter. Peter: $x - 3$ Sonia: $2(x - 3)$
Equations	22 23	Solve linear equations and use to solve problems These may have: a single denominator, fractional and negative solutions, brackets, variable on both sides	$3a + 2 = 14$ $5a - 1 = 8$ $9a + 2 = 5a + 30$ $\frac{a + 3}{5} = 4$ $9(2a + 1) = 27$ $\frac{2}{3}a - 1 = 9$
Formulae	22 23	Use of known and given formulae. Drawing a line graph from a real-world formula	$s = u + at$ Find the value of s when $u = -5$, $a = 9.8$ and $t = 12$

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
		Algebra	
Straight line graphs	26 27 28 29	In the form: $y = \pm a$, $x = \pm a$, $y = \pm x$ $y = mx + c$ Gradient and intercept **not** required but own table of values or similar may be needed	Draw and label the lines $y = -5$ and $y = 3x - 2$
Sequences	31 32 33	Sequences may be arithmetic or geometric n^{th} term will not be tested explicitly, but may be useful Consideration of square numbers, triangular numbers, Fibonacci, etc.	5, 8, 11, 14, … Find next term. Find 20^{th} term. A sequence is made by x 3 then +1 from previous term. Find 3^{rd} term. Find 1^{st} term.

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
		Measurement	
Metric units	36	Knowledge, conversion and problems involving: mm, cm, m, km g, kg *l*, ml s Knowledge and problems involving °C, square and cubic units	
Other units	*36*	Knowledge, conversion and problems involving £, p, hours, minutes Some knowledge of °F, miles, feet, inches	
Time		Knowledge of 12-hour and 24-hour clock Calculating time and using timetables Relationship between times in hours expressed as mixed numbers or decimals and hours and minutes	Midnight 00:00 12:00 a.m. A train leaves at 11:38 and travels for $1\frac{3}{4}$ hours. When does it arrive?

Pupils should have knowledge and understanding of:		Further guidance	Examples *(not an exhaustive list)*
NC	**NC**	**Measurement**	
Money	12 43 45	Knowledge of £ and p and common UK coins and notes Problems involving calculations with and without a calculator	
Speed, distance and time	45	Understanding of units of speed: km/h, m/s, mph Use of formulae Answers may be expressed in hours and minutes Some simple, mental methods for speed relating to 15, 20 or 30 minutes	$T = \frac{D}{S} = \frac{120\text{km}}{80\text{km/h}} = 1\frac{1}{2}$ hours $= 1$ hour 30 minutes 3 miles in 15 minutes Speed = 4 × 3 = 12 mph
Area	46	Area of rectangle, triangle, parallelogram, trapezium and composite shapes	$A = \frac{b \times h}{2}$
Perimeter	47	Perimeter of common shapes and composite shapes	
Pi	46 47	Area and circumference of circles Area and perimeter of sectors and composite shapes	$A = \pi r^2$ $C = \pi D$
Solids, nets, volume Isometric drawing	60	Faces, edges, vertices Draw net of cuboid or prism on a grid Find surface area and/or volume of cuboid Draw cuboids on isometric grid	$V = l \times w \times h$
Real world line graphs	68	Including conversion graphs, scatter graphs, travel graphs Draw, read off value and make predictions Reading from a curve may be required (but **not** drawing of curve)	

Pupils should have knowledge and understanding of:		Further guidance	Examples *(not an exhaustive list)*
NC	**NC**	**Geometry**	
Geometric terminology and symbols	50 51 52	Parallel, perpendicular, right-angle, equal, regular, irregular, equilateral, isosceles, similar, congruent Point, line and angle notation: PQ, $\angle PQR$	
Measure and draw angles and lines	*52*	Measure and draw line segments to nearest mm Measure and draw angles to the nearest degree	
2D shapes	50	Names and properties of polygons: triangle though to decagon Quadrilaterals: (isosceles) trapezium, parallelogram, kite, rhombus, rectangle, square Triangles: isosceles, equilateral, right-angled, scalene	
Line symmetry **Rotational symmetry**	50	Recognise lines of symmetry and draw Recognise order of rotational symmetry and draw	
Coordinates	25	In all 4 quadrants, usually integer values	
Transformations on coordinate grid	53 55	Translation (in the form 3 units to the left, 1 unit down), reflection, rotation, enlargement (by positive integer) Use of linear and area scale factor	
Angle calculations	55 56	Vertically opposite angles, angles on a straight line, angles at a point, alternate and corresponding angles, and angles in triangles Multi-step angle chasing problems	
Angles in polygons	57 58	Finding exterior, interior and related angles Finding the sum of the interior angles Knowledge of triangle: 180°, quadrilateral: 360° Finding the number of sides of a regular polygon	

Pupils should have knowledge and understanding of:		Further guidance	Examples *(not an exhaustive list)*
NC	**NC**	**Probability and Statistics**	
Probability	62 63 64	Understanding probability ranges from 0 to 1 (impossible to certain) Expressing probabilities as a fraction in lowest terms Listing all possible outcomes Finding likely number of events from a probability Some knowledge of dice and playing cards expected	p(red) = $\frac{8}{12}$ = $\frac{2}{3}$ H H H T H H H H T T H T H T H T T H H T T T T T
Averages	*66*	Finding mean, median, mode and range of a set of data Understanding usefulness of types of average Using mean to find total and next value	5 , 12 , 7 , 8 , 7 , 9 mean = $\frac{48}{6}$ = 8 mode = 7 5, 7, 7, 8, 9, 12 median = 7.5 range = 12 − 5 = 7
Frequency tables, bar charts, pie charts		Construct bar chart from a frequency table Interpret bar chart or pie chart	

FURTHER TOPICS FOR ADDITIONAL LEVEL

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
Pythagoras' Theorem	59 60		
Further algebra			
• **Rearranging formulae**	22		$5a + 2b = 39$ $3a - b = 19$
• **Simultaneous equations**	25 30	2 equations and 2 variables, may be in context solved algebraically or graphically	
• **Equations with more than one denominator**	21		$\frac{2x}{3} + \frac{x-3}{4} = 5$
• **Multiplying out brackets by a variable**			$5x(2x-3) - x(x+1)$
• **Factorising by a variable**		May be required for further simplification or rearranging of formulae	$5x^2 + x$
• **Quadratic equations**	35	Simple examples or solved graphically	$x^2 = 9$ $\quad x(x+2) = 2x + 16$
• **Algebraic proof**		May be asked to 'show that …' Candidates should use ■ (a box) or Q.E.D. to conclude a proof	$\underline{\underline{x = \pm 3}}$ $\quad x^2 + 2x = 2x + 16$ $x^2 = 16$ $\underline{\underline{x = \pm 4}}$
Plotting curves	27 31	May be parabolic or reciprocal May require own table of values or similar May be used for graphical solutions to equations	$y = 10 - (x+1)^2$ $y = \frac{5}{x}$
Volume and surface area of prisms and cylinders	46		
Reverse percentages	43	Finding the original value	

FURTHER TOPICS FOR ADDITIONAL LEVEL

Pupils should have knowledge and understanding of:	NC	Further guidance	Examples *(not an exhaustive list)*
n^{th} term of sequences	32	May be tested explicitly	
Multi-part journeys	45	Find average speed over multi-part journeys	
Enlargement by fractional scale factor	54		
HCF and LCM	3	May be tested explicitly	
Significant figures	13		
Standard form	8	With positive indices	

MENTAL ARITHMETIC: USEFUL STRATEGIES

NC

Addition and Subtraction

Commutative and not commutative

$3 + 5 = 5 + 3$ $(10 - 3 \neq 3 - 10)$

Finding 10's complements, etc.

$8 + 5 + 2 + 5$
$= 10 + 10$

$23 + 94 - 3 + 106$
$= 20 + 200$

Using near multiples of 10, etc.

$19 + 32$
$= 20 + 30 + 1$

Relationship to multiplication

$7 + 7 + 7 + 7 + 7$
$= 7 \times 5$

$7 + 8 + 9$
$= 8 \times 3$

Subtraction by counting on

$57 - 48$ (but understanding of the order must be maintained: not $48 - 57$)
$= 7 + 2$

Use of place value

$360 - 123$ subtract 100, subtract 20, subtract 3: $360 \rightarrow 260 \rightarrow 240 \rightarrow 237$

$40 + 200 + 3$

Multiplication and Division

Extending from known times tables

$7 \times 12 \rightarrow 7 \times 13$
$7 \times 12 \rightarrow 7 \times 24$
$4 \times 3 \rightarrow 40 \times 30$
$3 \times 9 \rightarrow 270 \div 90$
$6 \times 7 \rightarrow 4.2 \div 7$

Finding multiples of 10, etc.

$2 \times 17 \times 5$
$= 10 \times 17$

Halving and doubling, etc.

5×14
$= 10 \times 7$

15×16
$= 30 \times 8$

28×25
$= 7 \times 100$

WHAT'S OUT AND WHAT'S IN?

A quick reference to some of the major changes

What's OUT	
From the Core & Foundation Level	
• Bearings	Map scale drawings will **not** be examined, but candidates will still be expected to know compass directions
• Constructions	Accurate drawing will **not** be examined, but candidates will still be expected to measure lengths and angles accurately
• Drawing of pie charts	Candidates will **not** be expected to draw pie charts but will still be expected to interpret them
• Significant figures	Candidates will be specifically asked to round to nearest whole number, multiples of 10 or decimal place. Estimation may involve 'sensible' rounding
• Determining averages from frequency tables	Frequency tables will be used, and mean, median and mode will be examined, but **not** from data extracted from a frequency table
From the Additional Level	
• All the above from the Core Level **except significant figures**	
• Solving inequalities	

What's IN	
From ALL levels	
• Written mental arithmetic	Tested in a new-style Mental Arithmetic paper with written questions rather than aural
• Increased testing of core skills (with fewer words)	
• Writing units in answers	
• Questions with less scaffolding	Candidates will less often be led through problems, step by step
• An increased number of thought-provoking puzzles	More unfamiliar problems may be presented towards the end of papers to encourage candidates to explore and experiment
• Expectation of greater quality of thought and working, rather than quantity of questions	

NATIONAL CURRICULUM

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	NUMBER		
1	understand and use place value for decimals, measures and integers of any size	✓	✓
2	order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥	= , < , > only	✓
3	use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property	HCF and LCM terminology not expected	✓
4	use the 4 operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative	mixed number arithmetic will not be examined, but answers may be required as mixed numbers	✓
5	use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals	✓	✓
6	recognise and use relationships between operations including inverse operations	✓	✓
7	use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations	powers may go above 5 in puzzle questions	✓
8	interpret and compare numbers in standard form $A \times 10^n$ $1 \leq A < 10$, where n is a positive or negative integer or 0	not examined	using positive indices only
9	work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)	✓	✓
10	define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express 1 quantity as a percentage of another, compare 2 quantities using percentages, and work with percentages greater than 100%	percentages will be less than 100%	✓

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	NUMBER *(continued)*		
11	interpret fractions and percentages as operators	✓	✓
12	use standard units of mass, length, time, money and other measures, including with decimal quantities	✓	✓
13	round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]	significant figures will not be examined	✓
14	use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation $a<x\leq b$	errors expressed using inequality notation $a < x \leq b$ will not be examined	
15	use a calculator and other technologies to calculate results accurately and then interpret them appropriately	✓	✓
16	appreciate the infinite nature of the sets of integers, real and rational numbers	not examined	✓
	ALGEBRA		
17	use and interpret algebraic notation, including:		
	• ab in place of $a \times b$	✓	✓
	• $3y$ in place of $y + y + y$ and $3 \times y$	✓	✓
	• a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$;	✓	✓
	• a^2b in place of $a \times a \times b$	✓	✓
	• $\frac{a}{b}$ in place of $a \div b$	✓	✓
	• coefficients written as fractions rather than as decimals	✓	✓
	• brackets	✓	✓
18	substitute numerical values into formulae and expressions, including scientific formulae	knowledge and understanding of scientific formulae not expected	
19	understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors	solving inequalities not examined	
20	simplify and manipulate algebraic expressions to maintain equivalence by:		
	• collecting like terms	✓	✓
	• multiplying a single term over a bracket	only number outside the bracket	may have variable outside bracket
	• taking out common factors	factor restricted to an integer	✓
	• dividing an expression by an integer or by another expression	✓	✓
	• expanding products of 2 or more binomials	not examined	

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	ALGEBRA *(continued)*		
21	understand and use standard mathematical formulae; rearrange formulae to change the subject	rearranging formulae not expected but may be useful	✓
22	model situations or procedures by translating them into algebraic expressions or formulae and by using graphs	✓	may include simultaneous equations
23	use algebraic methods to solve linear equations in 1 variable (including all forms which require rearrangement)	✓	✓
24	use algebraic methods to solve linear equations in 2 variables (including all forms which require rearrangement)	not examined	✓
25	work with coordinates in all 4 quadrants	✓	✓
26	recognise, sketch and produce graphs of linear and quadratic functions of 1 variable with appropriate scaling, using equations in x and y and the Cartesian plane	functions will be linear in the form y = ...	functions may require rearranging and may be quadratic
27	interpret mathematical relationships both algebraically and graphically	✓	✓
28	reduce a given linear equation in two variables to the standard form $y = mx + c$; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically	y intercept and gradient will not be examined	
29	use linear and quadratic graphs to estimate values of y for given values of x and vice versa and to find approximate solutions of simultaneous linear equations	graphical solutions to simultaneous equations will not be examined	graphical solutions to simultaneous equations may be examined
30	find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs	✓	✓
31	generate terms of a sequence either from a term-to-term or a position-to-term rule	✓	✓
32	recognise arithmetic sequences and find the n^{th} term	n^{th} term will not be explicitly examined	n^{th} term may be examined explicitly
33	recognise geometric sequences and appreciate other sequences which arise	✓	✓
34	solving simple linear inequalities	not examined	
35	solving simple quadratic equations	not examined	equations such as $x^2 = 9$ *or* $x(x + 2) = 2x + 16$

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	RATIO, PROPORTION and RATES OF CHANGE		
36	change freely between related standard units (for example time, length, area, volume/ capacity, mass)	✓	✓
37	use scale factors, scale diagrams and maps	map scales and bearings will not be examined	
38	express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1	✓	✓
39	use ratio notation, including reduction to simplest form	✓	✓
40	divide a given quantity into 2 parts in a given part:part or part:whole ratio; express the division of a quantity into 2 parts as a ratio	ratios may have up to 3 parts	
41	understand that a multiplicative relationship between 2 quantities can be expressed as a ratio or a fraction	✓	✓
42	relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions	✓	✓
43	solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics	reverse percentages to find original value will not be examined	✓
44	solve problems involving direct and inverse proportion, including graphical representations	inverse proportion will not be examined	✓
45	use compound units such as speed, unit pricing and density to solve problems	knowledge of the term *density* will not be expected	average speed for multi-part journeys may be examined knowledge of the term *density* will not be expected
	GEOMETRY and MEASURES		
46	derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)	volume of prisms other than cuboids will not be examined candidates not expected to derive formulae	✓
47	calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes	radius or diameter will be given	candidates may be expected to find radius or diameter
48	draw and measure line segments and angles in geometric figures, including interpreting scale drawings	draw and measure line segments and angles only in the examination, candidates will not be expected to draw accurately plane figures or scale drawings	

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	GEOMETRY and MEASURES *(continued)*		
49	derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line	accurate drawing will not be examined recognising the perpendicular distance from a point to a line as the shortest distance may be useful	
50	describe, sketch and draw, using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons which are reflectively and rotationally symmetric	regular polygons will be limited to those whose exterior angle is an integral number of degrees names of polygons up to decagons will be expected	✓
51	use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles	✓	✓
52	derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies	✓	✓
53	identify properties of, and describe the results of, translations, rotations and reflections applied to given figures	vector terminology will not be examined; translations will be described in terms of units moved left/right and up/down	
54	identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids	construction of triangles will not be examined enlargements will be by positive whole numbers linear and area scale factor may be examined	enlargements may be by a positive fractional scale factor candidates may be required to deduce the scale factor and centre of enlargement
55	apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles	✓	✓
56	understand and use the relationship between parallel lines and alternate and corresponding angles	✓	✓
57	derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons	✓	✓

	National Curriculum Descriptors Pupils should be taught to:	Core Level	Additional Level
	GEOMETRY and MEASURES *(continued)*		
58	apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs	Pythagoras' Theorem will not be examined proofs will not be examined	Pythagoras' Theorem may be examined but not derived
59	use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles	Pythagoras' Theorem will not be examined trigonometry will not be examined	Pythagoras' Theorem may be examined trigonometry will not be examined
60	use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D	limited to cubes, cuboids, pyramids and prisms	limited to cubes, cuboids, pyramids, prisms and cylinders
61	interpret mathematical relationships both algebraically and geometrically	✓	✓
	PROBABILITY and STATISTICS		
62	record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale	✓	✓
63	understand that the probabilities of all possible outcomes sum to 1	✓	✓
64	enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams	Venn diagrams will not be examined explicitly	
65	generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities	combined events may be listed in a table	
66	describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)	✓	✓
67	construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data	candidates may be asked to interpret pie charts but not to draw them	
68	describe simple mathematical relationships between 2 variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs	✓	✓