Thinking Mathematically

Here is a rectangle divided into squares.

How many squares are here altogether?*



This question, from a 13+ Common Entrance paper, is a simpler version of a problem that asks, 'How many squares are there on a chessboard?'

There are many problem solving strategies it helps to be familiar with. One approach is to **simplify the problem** and **look for a pattern**.

Rather than starting with an 8x8 grid of squares, begin instead with a 1x1 grid, then consider a 2x2 grid, then a 3x3 grid and so on, **systematically** recording the number of 1x1 squares, the number of 2x2 squares, the number of 3x3 squares etc. for each grid so that you can easily spot any patterns.

When solving problems, sooner or later you are likely to become stuck. Getting **unstuck** is therefore an essential part of the problem solving process. As well as being **curious**, Mathematicians must be **tenacious** and **resilient**, and perhaps even a little bit **courageous**.

Devising an efficient strategy requires **creative thinking**. Then, once you think you have solved a problem, **critical thinking** will help you to evaluate if your solution is correct (as well as if you have found all possible solutions).

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience and enjoy the triumph of discover.

from How to Solve It by George Pólya

Simplify the problem and look for a pattern: a few more problems to keep you busy!

- 1. If there were eighteen pupils in a class and each pupil shakes hands just once with everyone else there, how many handshakes were there altogether?
- 2. How many diagonals does a regular decagon have?
- 3. Show that $3^{2022} \div 4$ has a remainder of 1.

*Question from CE at 13+ CORE MATHEMATICS Specimen Calculator Paper